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Is Sustainable Growth Optimal?

ANDREA BARANZINI
University of Geneva, Department of Economics, CUEPE and IAE, 102 bld Carl-Vogt, CH-1211 Genève 4, Switzerland

FRANCOIS BOURGUIGNON
DELTA-Département et Laboratoire d'Economie Appliquée, Ecole Normale Supérieure, 48 bld Jourdan, 75014 Paris, France

Abstract

This paper analyzes sustainable growth in a stochastic environment, with human extinction as a possible outcome. The basic constraint of sustainability is that consumption never decreases over an infinite horizon, which requires that the probability of extinction be maintained at zero. We show that this problem can be examined in a standard optimal-growth model. Under certain conditions, the solution of this problem is a corner solution with probability of survival equal to one, at the cost of economic growth. These conditions depend on the initial development level and on the elasticity of utility with respect to consumption. In some circumstances, which depend on the social discount rate, optimal-growth paths do not exist. In these situations, the sustainable-growth concept has a clear autonomy with respect to the usual optimality criterion.

Key words: sustainable development, extinction, optimal endogenous growth

1. Introduction

Since its introduction by the World Commission on Environment and Development (WCED, 1987), sustainable growth as a concept has been gaining ground in the literature on development, natural resources, and environment. Yet this concept has been rather ambiguously defined. For some authors, economic growth is sustainable if it can proceed ad infinitum at a constant rate consistent with the availability or the renewability of natural resources (Krautkraemer, 1990). Other authors consider a growth path sustainable if, given all natural constraints, it requires no sacrifices from future generations—that is, consumption per capita will never go down from one generation to the next (Pezzey, 1989; Tietenberg, 1988). Yet another group views sustainable economic growth as simply the level of human economic activity which, given the technology available now and presumably in the future, leaves the environment unchanged—including animal and vegetal species and the availability of some basic natural resources (Pearce, Markandya, and Barbier, 1988; Daly, 1990). Depending on the definition used, on the constraints actually taken into account, and on the assumptions made concerning the future evolution of technology, recommendations by the supporters of sustainable growth are—not surprisingly—imprecise and sometimes contradictory.

What is more surprising still is the lack of reference to the standard models of optimal growth theory. As stressed by Dasgupta (1993), sustainable growth is essentially an intertemporal consumption problem. As such, it should have something to do with that branch of economic theory that deals with the intertemporal allocation of production and consumption. Yet the sustainable growth literature borrows little from optimal-growth theory. Sustainable growth seems to be regarded as if it were in some sense an alternative and in opposition to optimal growth and to the neoclassical framework (cf. Barbier, 1987). The reason may be that most of the models that analyze sustainability issues in neoclassical terms—from the seminal work by Keeler, Spence, and Zeckauser (1971), Forster (1973, 1980), or Dasgupta and Heal (1974), to the more recent contributions of Pezzey (1989), Tahvonen (1991), or van der Ploeg and Withagen (1991)—are deterministic and thus miss one of the fundamental issues of sustainability: that of risk, the risk of human species extinction, in particular.

This paper approaches the problem by attempting to explicitly account for the probability of survival in a conventional optimal stochastic growth model. By analyzing how the optimal-growth policy changes with parameters such as the degree of risk aversion or the time discount rate, we should be in a position to judge whether the general recommendations made by advocates of sustainable growth correspond to those resulting from standard optimal-growth models with Von Neumann-Morgenstern objective functions, or whether there is something truly peculiar in the objective function of sustainable growth. In the first case, the next task would be to analyze how the applications of optimal growth theory to environmental and natural resource issues can be generalized, when the explicit probability of extinction is taken into account. In the second case, some thought would need to be given to the reasons for the apparent suboptimality of the sustainable-growth objective.

The general optimal stochastic growth model with explicit and partly controllable probability of survival is difficult to solve. This paper analyzes a simpler model. At some point in time, society is faced with the following technological decision: whether to adopt or not a new technology that will raise the rate of GDP growth by some variable amount. However, this will also raise the instantaneous hazard rate of extinction at some time in the future. In such a framework, sustainability would require minimizing the risk of extinction and therefore the rate of innovation and economic growth. Under what conditions set on the parameters of the model would optimal growth lead to the same recommendation? Are these conditions plausible?

Our model is rather simple. As a matter of fact, it is close to some models used in the economics of health (Grossman, 1972; Ehrlich and Chuma, 1990). This is not surprising, as an obvious analogy exists between someone controlling their longevity through investments in health, which reduce the growth of personal wealth, and a society controlling the advent of major environmental risks, at the cost of economic growth. There is a big difference between the two problems, however. Expected optimal longevity might be infinite (or hoped to be so) in the case of a society with sufficient intergenerational altruism. It is, however, finite for individuals—even for those whose discount rate is extremely low and initial endowments in health and wealth are extremely large. This difference explains why, in what follows, we generally insist on the set of conditions guaranteeing that society will try to eliminate all risks of environmental catastrophe and will optimally have an infinite lifetime, rather than insisting what the optimal lifetime should be when it is finite.

The paper is organized as follows. Section 2 discusses the definition of sustainable growth in connection with optimal-growth problems including some uncertainty about the survival of society. In its most general form, the corresponding optimal stochastic problem, unfortunately, is rather difficult to analyze and is intractable. Thus, we propose a slightly different form of the original problem, explicitly based on the probability of survival. We analyze a simple specification of that model in some detail in Section 3. Several interesting results are derived, which show that the sustainable-growth view is consistent with the optimal-growth framework in cases that depend on the elasticity of utility with respect to consumption, the time discount rate, some technological characteristics, and, more interestingly, the initial stock of resources relative to minimum consumption. Sustainable growth thus appears to be optimal in economies that have already reached some affluence.

2. Sustainable growth in an optimal-growth framework

First we define analytically what is meant by sustainable growth. Various definitions have been offered that are not exclusive of each other. In what follows, we shall stick to what seems to be the most general and at the same time the most widely accepted definition. Following Pearce, Markandya, and Barbier (1988) and Pezzey (1989), we shall say that *development is sustainable if the welfare of a society—or its social achievements—does not decrease over time*. In theory, the welfare of a society should be defined on a vector of economic and social variables: real consumption per capita, income distribution, educational achievements, health and nutritional status, and so on. In the aggregate framework we plan to use here, this vector will be reduced to consumption per capita. This is not overly restrictive.

Taken literally, the preceding definition seems to be trivial. The conditions under which consumption per capita is nondecreasing in the conventional optimal growth model are widely known. It is sufficient that the discount rate be small enough. However, the situation is much more complex when certain productive factors (such as some natural resources) are exhaustible and cannot be produced. Consumption per capita might well be nondecreasing as long as certain natural resources are available, but it might then drop dramatically to zero, implying the extinction of the human species. In that case, sustainable growth clearly requires a natural-resource conservation policy that might be at odds with what would be recommended by a standard optimal growth model. The definition used above of sustainable growth is thus operational.

The definition is slightly incomplete when some uncertainty is introduced into the model. Suppose that output per capita is subject to some uncertainty. Then, optimal (stochastic) growth might well involve some strictly positive probability of a drop in consumption per capita, the value of that probability depending presumably on the degree of social risk aversion. *We take it that sustainable growth in such a context requires that under no circumstance can consumption decrease over an infinite horizon.*[1] One could easily imagine situations in which this requirement would significantly alter the growth policy resulting from an optimal growth model. In particular, it is clear that sustainable growth should have a strong bias against all kinds of environmental risks.

In standard optimal-growth models, no reason exists a priori to rule out the possibility that the time horizon might optimally be finite with some strictly positive probability and that consumption might decrease at some points in time. In our view, sustainable growth, on the contrary, would require both events to occur with a zero probability—if physically possible. The problem is to know whether both points of view can be consistent with each other. A simple quasi-deterministic model should provide us with some notion of these properties. This model will be used in the rest of this paper.

Let $P(t)$ be the probability that society will be alive at time t, and the associated hazard rate be λ. This rate is such that

$$\dot{P}(t) = -P\lambda. \tag{1}$$

λ may depend negatively on the stock of resources per capita, k, so that the instantaneous probability of extinction increases with the depletion of resources.[2] The hazard rate, however, may also depend directly on the flow of consumption, c. A general representation of this process is the following:

$$P(t) = e^{-\Lambda};\ \dot{\Lambda} = \lambda = \phi(k, c, A, \Lambda), \tag{2}$$

where A is some control variable, such as a technological choice or the level of abatement activities; $\phi_k \leq 0$ and $\phi_c \geq 0$. The survival of society thus depends on a stock variable, Λ, the accumulation of which depends on the flow of consumption in relation to the stock of resources and on some other parameter under the control of the economy. In this framework, Λ may be interpreted either as a stock of pollution or, equivalently, as the opposite of a stock of natural resources.

Given that representation of the survival or extinction process, the optimal-growth model may be formulated in the following terms. The objective is to maximize the flow of expected utility over an infinite horizon. Expected utility at instant t is simply the discounted utility of consumption $u(c) \cdot e^{-\delta t}$, weighted by the probability of survival $P(t)$:[3]

$$\underset{(c,A)}{\text{Max}} \int_0^\infty u(c)e^{-[\delta t+\Lambda]}dt. \tag{3}$$

The constraints bear on the stock of resources, k, and the survival potential, Λ:

$$\begin{aligned} \dot{k} &= f(A)k - c, \\ \dot{\Lambda} &= \phi(k, c, A, \Lambda), \\ k(0) &= k_0, \quad \Lambda(0) = \Lambda_0, \end{aligned} \tag{4}$$

where the production technology is assumed to depend also on the control variable, A, entering the hazard function $\phi(\cdot)$. This feature is a simple way of taking into account the fundamental externality between technological choices or abatement activities and environmental risk. In the present case, it is common to assume that both $f(\cdot)$ and $\phi(\cdot)$ are increasing (or both decreasing) functions of A. Under these conditions, reducing the hazard rate by reducing A clearly has a direct production cost or, similarly, increasing the productivity of the resources stock, k, has direct negative consequences on future survival probabilities.

This model is conceptually extremely close to some canonical models in environmental literature—for instance, van der Ploeg and Withagen (1991).[4] The analytical advantage of the model is that it is deterministic. Its disadvantage is that it relies on two state variables, k and Λ, rather than on one.

In this modified framework, the sustainable-growth objective might be expressed as follows: (1) the greatest probability of survival at all times and (2) nondecreasing consumption, conditional on survival.

The problem is to know under what conditions, if any, on the utility function $u(\cdot)$, on the hazard function $\phi(\cdot)$, on the discount rate δ, on the marginal productivity of technology $f'(A)$, and on initial conditions, the solution of the optimal growth model (3) and (4) satisfies the preceding sustainable-growth requirements. It is also important to note that nondecreasing consumption is conditional on survival and it is not on expected value, as in Heal (1982).

3. Optimal sustainable growth in a simple model

This section will analyze the simple growth model (3) and (4). However, to keep things as simple as possible, we assume a hyperbolic absolute risk aversion (HARA) utility function and linear functions for $f(\cdot)$ and $\phi(\cdot)$. The modified problem is then close to the familiar life-cycle (infinite horizon) model or to the optimal (endogenous) growth model:[5]

$$\begin{aligned} &\underset{(c,A)}{\text{Max}} \int_0^\infty u(c)e^{-\delta t-\Lambda}dt, \\ &\text{s.t.:} \quad \dot{k} = (\mu + Ap)k - c, \\ &\qquad \dot{\Lambda} = Am - b\Lambda, \\ &\qquad k(0) = k_0 \text{ and } \Lambda(0) = \Lambda_0, \\ &\qquad k(t) \geq 0 \text{ and } A \geq 0\ \forall t, \end{aligned} \tag{5}$$

where $A \in [0, \bar{A}]$, μ is the autonomous rate of increase of k, and m, b, and p are positive constants that may be interpreted, respectively, as the effect that the technological choice has on the hazard rate, the rate at which that effect declines naturally over time, and the marginal productivity of technology. Thus, the hazard-rate function is assumed to depend neither on the stock of resources nor on current consumption. Those restrictions lead to relatively simple analytical results, while conveying the essence of the problem under analysis.

The HARA utility function we adopt is

$$u(c) = \frac{1-\gamma}{\gamma}\left(\frac{c - C}{1-\gamma}\right)^{\gamma}, \tag{6}$$

where C is a minimum consumption level. Indeed, if it were possible to live on nothing, the issue of sustainable growth would lose its importance. We also impose $\gamma < 1$ for concavity.[6]

The interpretation of parameter γ is multiple: it represents relative risk aversion, the inverse of intertemporal elasticity of substitution, and the instantaneous elasticity of utility with respect to consumption.[7] However, in our context, it is the role of γ as the instantaneous elasticity of utility that is the correct interpretation.

We distinguish two cases that depend on the value of the γ parameter of the HARA specification:

1. $\gamma \in (0, 1)$. In this case, utility is unbounded and tends toward zero when c approaches its minimum, C. Thus utility is always positive.[8]
2. $\gamma \leq 0$. Here utility is bounded by zero and tends toward minus infinity when c approaches its lower bound C. Utility is then always negative.

Because of the important differences between these two cases, we handle them separately, beginning with the case of positive utility functions.

To find the solution to problem (5), we first solve the problem where A is taken to be constant over time, as if some irreversible technological choice had to be made at time zero.[9] We take the initial value of Λ and the depreciation rate b to be zero.[10] We then come back to the original problem, where A may vary over time and consider the case where Λ_0 and b are strictly positive.

3.1. *The case of irreversible technological choice (A constant) when $\gamma \in (0, 1)$*

We solve (5) in three steps. We first look for the optimal consumption path consistent with a given value of A. We then evaluate the corresponding value of the objective function $W^*(A)$. Finally, we maximize that expression with respect to the control variable A.

In order to find the optimal path of consumption, we first notice that introducing an exponential probability of survival is equivalent to modifying the time discount rate from δ to $\delta + Am$. With that discount rate, and using the maximum principle, the optimal path for consumption is given by

$$c^* = C + (1 - \gamma)\psi_0^{1/(\gamma-1)}e^{t(\mu+Ap-\delta-Am)/(1-\gamma)}, \tag{7}$$

where ψ_0 is the initial shadow price of capital.

Therefore, optimal consumption is minimal consumption plus an exponential factor that grows at the rate

$$\theta = [(\mu + Ap) - (\delta + Am)]/(1 - \gamma), \tag{8}$$

where θ is the asymptotic optimal rate of growth of consumption. As usual, this rate is simply the difference between the productivity of capital and the (modified) discount rate, weighted by the intertemporal elasticity of substitution. As defined by the sustainable objective (2) used above, sustainable growth requires nondecreasing consumption, thus $\theta > 0$.

The shadow price of capital in the initial period is found by integrating the capital movement equation in (5) and using the transversality condition

$$\lim_{t\to\infty} \psi k e^{-(\delta+Am)t} = 0. \tag{9}$$

Assumption 1:

$$\alpha = \frac{(\delta - \mu\gamma)}{(1 - \gamma)} + A\frac{(m - p\gamma)}{(1 - \gamma)} = (\mu + Ap) - \theta > 0, \tag{10}$$

$$k_0 > \frac{C}{(\mu + Ap)} \tag{11}$$

Under Assumption 1, the initial shadow price of capital is given by

$$\psi_0^{1/(\gamma-1)} = \frac{\alpha}{(1 - \gamma)}\left[k_0 - \frac{C}{(\mu + Ap)}\right] \tag{12}$$

Not surprisingly, the initial shadow price of capital, or equivalently the initial level of consumption, depends on the discrepancy α between the rate of growth permitted by technology, $\mu + Ap$, and the optimal growth rate of consumption, θ in (8). In addition, both variables also depend on the initial volume of capital in excess of what is necessary to sustain the minimum level of consumption forever.

If (10) is not satisfied, then no known solution exists to the optimal growth model (5), because the objective function is divergent.[11] This implies serious bounds for the parameters of the model. In the case of special interest, $A = 0$, it must be true in particular that

$$\delta > \mu\gamma. \tag{13}$$

In other words, the discount rate cannot be too small in comparison with the productivity of capital and the elasticity of utility with respect to consumption.[12] More generally, this condition also implies bounds on the value of A, but these are irrelevant in what follows, since we essentially concentrate on the optimality conditions for $A = 0$.

Clearly, the preceding assumption does not hold (for $A = 0$) if the discount rate δ is zero or close to zero. This has been the subject of controversy in the optimal-growth literature, especially in connection with environmental issues (see, for instance, Parfit, 1984; Pearce, Barbier, and Markandya, 1988; Pearce, Markandya, and Barbier, 1989). We return to that problem when dealing with the case $\gamma < 0$ (see Section 3.3).

The assumption in (11) is not controversial. Clearly, sustainable growth above (or at) the minimum consumption level C requires some minimum initial capital stock.

Under Assumption 1 and given (12), it is easy to derive the welfare of society. Substituting (7) and (12) in the objective function, yields

$$W^* = \frac{(1 - \gamma)^{1-\gamma}}{\gamma}\alpha^{\gamma-1}\left[k_0 - \frac{C}{\mu + Ap}\right]^{\gamma}. \tag{14}$$

Total utility thus appears as a function of the initial capital stock, in excess of what is required to satisfy minimum consumption forever, and of the discrepancy, α, between the maximum rate of growth permitted by technology and the optimal growth rate of consumption.

We now consider the second step of the overall optimization procedure, which is concerned with the value of the irreversible technological parameter, A. Following the sustainable growth objective used above, we have

Definition. *Sustainable growth is defined by the minimum environmental risk* $A = 0$ *and nondecreasing consumption.*

In what follows, we thus discuss the conditions under which the maximization of social welfare, W^* in (14), requires A to be zero. It is easily established that the derivative of W^* with respect to A is given by

$$\frac{1}{W^*}\frac{\partial W^*}{\partial A} = -\frac{(m - p\gamma)}{\alpha} + \gamma \frac{Cp}{(\mu + Ap)^2}\left[k_0 - \frac{C}{(\mu + Ap)}\right]^{-1} \tag{15}$$

We thus get the following propositions:

Proposition 1.a. *Under Assumption 1, technological irreversibility and* $\gamma \in (0, 1)$, *a necessary condition for sustainable growth to be optimal is that*

$$\delta \leq \mu \text{ and } \frac{m}{p} > \gamma + \frac{\gamma\alpha_0 C}{\mu k_0 - C} \tag{16}$$

with

$$\alpha_0 = \frac{\delta - \mu\gamma}{1 - \gamma}$$

Proposition 1.b. *Under the assumptions of the preceding proposition, a sufficient condition for sustainable growth to be optimal is* (16) *together with*

$$\mu\frac{k_0}{C} > \frac{\gamma}{2(1 - \gamma)} \tag{17}$$

It follows that (16) *is both necessary and sufficient if* $\gamma \in]0, 1/2]$.

The intuition behind (16) is essentially that sustainable growth is optimal when the environmental risk of technology, m, is larger than its effect, p, on the productivity of capital in a proportion that depends on the elasticity γ of utility. Indeed, assume that technology is increased by dA, then consumption may be increased at all times in a proportion pdA and utility in a proportion γpdA. At the same time, however, expected utility decreases in the proportion mdA because of increased risk. No innovation is thus to be undertaken if $m/p > \gamma$, which is the first part of (16).

The corrective term in (16) essentially involves the minimum consumption standard C. It accounts for the fact that the elasticity of the utility function is γ only asymptotically and depends on $(c - C)$ when consumption is close to its minimum standard.

Condition (17), which is a sufficient condition, is stricter than (16), because it requires a higher k_0/C. Indeed, the larger the initial capital stock, with respect to what is needed for subsistence, the less additional utility is obtained from productivity gains—the second term in (15)—and thus the more likely the risk aspect of technological innovation dominates.

Several remarks can be made on the basis of the preceding arguments.

First, note that the coefficient γ in the condition (16) for consistency between sustainable and optimal growth, with some transformation, could describe the intertemporal elasticity of substitution of society, or risk aversion. As shown by the previous static argument, however, it is really the definition of γ as the elasticity of absolute utility that matters here.[13] There is no real scope for intertemporal substitution, nor for risk aversion, in the irreversible choice of the technological parameter A.

Second, the most important aspect of Proposition 1 in the case of unbounded utility is that sustainable growth (as defined by the property that the maximum sacrifice is made in order to maintain the survival probability at one forever) clearly appears as a luxury good. Indeed, it is only when the stock of resources is above some minimum, which depends on the minimum survival consumption, C, that the extra growth permitted by technological innovation may be ignored. Below that minimum, society will find it optimal to adopt a fast-growth technology, even though that irreversible choice may be at the cost of a strictly positive extinction probability. From (16), this minimum capital stock is given by

$$k_0 > \frac{C}{\mu}\left[1 + \frac{p\gamma\alpha_0}{(m - p\gamma)\mu}\right] \tag{18}$$

This result presents some analogy with the arguments developed by L. Summers when, as the head economist of the World Bank, he argued that environmental issues were less important for developing than for developed nations (Summers, 1992; *The Economist*, 1992). As stated by Grossman (1993), this does not mean that the poor have a greater tolerance for environmental risks or different tastes for the environment. Rather, our model points out that it is the low level of affluence that forces the poor to take more environmental risks. However, it may be seen that this property is directly linked to the hypothesis that there is some physical minimum standard of consumption, or more technically that the marginal utility of consumption for society becomes infinite before consumption is zero. That poorer economies tend to take more risk on their environment until they reach some minimum level of affluence would not appear in a model with the familiar isoelastic utility function.

Nevertheless, the relevance of the preceding result might be reduced because it holds only for a discount rate above some minimum. In our framework, if one follows the school of thought that opposes the use of such a discount rate on the basis of intergenerational equity, then the difference between a poor and a rich country might not exist. In that case, no well-defined optimal-growth path exists. This gives some intellectual autonomy to the issue of sustainable growth in opposition to optimal growth.

3.2. The case of reversible technological choice when $\gamma \in (0, 1)$

We now move to the more general situation in which the choice of A is reversible, and the natural decline rate b is positive. Starting from model (5), we ask whether the conditions under which it is optimal to have $A = 0$ at any time are different from conditions (16) and (17). To do so, we begin by writing the first-order conditions (FOC) corresponding to model (5), with A fully flexible. We then examine under what conditions the solution obtained above with $A = 0$ satisfies the FOC of the more general model.

Denoting ψ as the (current-valued) implicit price of resources, k, and ρ as the (current-valued) shadow price of survival probability, Λ, the following FOC and transversality conditions must be satisfied:

$$u'(c) = \psi e^{\Lambda}, \tag{19}$$

$$\psi kp + \rho m < 0, \tag{20}$$

$$\dot{\psi} = (\delta - \mu)\psi, \tag{21}$$

$$\dot{\rho} = (\delta + b)\rho + u(c)e^{-\Lambda}, \tag{22}$$

$$\dot{k} = \mu k - c, \tag{23}$$

$$\dot{\Lambda} = -b\Lambda, \tag{24}$$

$$\lim_{t\to\infty} k\psi e^{-\delta t} = 0;\ k(0) = k_0, \tag{25}$$

$$\lim_{t\to\infty} \Lambda\rho e^{-\delta t} = 0;\ \Lambda(0) = \Lambda_0. \tag{26}$$

From the above, we know the solution of these equations for c, k, and ψ when $\Lambda_0 = 0$. It is not difficult to generalize to the case where $\Lambda_0 > 0$. Then, we integrate the differential equation (22) in ρ, using the transversality condition (25). Finally, we determine the conditions under which (20) holds, which imply that A is optimally equal to zero. When Λ_0 approaches zero, this leads to the following result:

Proposition 2. *With Assumption 1, technological reversibility, zero initial environmental risk ($\Lambda_0 = 0$), and $\gamma \in]0, 1[$, a necessary condition for sustainable growth to be optimal is that*

$$\delta \le \mu \quad and \quad \frac{m}{p} > \gamma\left(1 + \frac{C/u}{k_0 - C/\mu}\right)\left(1 + \frac{b}{\alpha_0}\right) \tag{27}$$

Note that, unlike Proposition 1, the preceding condition is only a necessary condition. The reason is that the objective function of the dynamic programming problem (4) is not concave with respect to Λ, so that the first-order conditions (19) through (26) are not sufficient. Several local maxima may exist in the problem at hand, and the preceding condition may be interpreted as necessary and sufficient only for such a local optimum. In what follows we ignore the additional conditions that would ensure that $A = 0$ is indeed a global optimum.

It must also be noted that (27) is a necessary condition for sustainable growth at any stage of the development process, that is, with the current capital stock, k—provided that the current survival probability of the economy is equal to unity ($\Lambda = 0$). As we may expect that condition (27) will in fact be more restrictive for the case in which the survival probability is strictly positive ($\Lambda > 0$), the reversibility of technology seems to imply that a nonsustainable path with $A > 0$ can be optimal only *transitorily.* A rigorous proof of that point would, however, require that condition (27)—and its generalization to the case where $\Lambda > 0$—be a necessary and sufficient condition for optimal growth.[14]

Proposition 2 generalizes the preceding one. Remember that in the irreversible case we simplify in assuming that $b = 0$. When $b = 0$ it may be seen that condition (27) in Proposition 2 is stronger than (16). In other words, all other parameters being the same, $A = 0$ now requires higher values of the environmental risk parameter, m.[15] Of course, this was to be expected. Irreversibility requires most precaution to be taken or, equivalently, involves some kind of option price. The difference between (27) with $b = 0$ and (16) allows us to measure the value of that option.

Consider now Proposition 2 with b positive. In this case, for given productive resources k, the survival-risk ratio m/p is now more or less binding, depending on the decline rate b. Everything else equal, a sustainable growth policy where no gamble at all is allowed on the risk survival, becomes less likely when the natural decline rate increases.

As before, sustainable growth appears to be a luxury. However, it may be shown that the threshold on initial resources is now higher compared to the threshold when technological choices were irreversible.

The preceding argument assumed that the economy was initially in a situation in which the survival probability was equal to one, or in other words a perfectly clean environment. All the arguments may easily be adapted to the case in which the initial value of Λ is not zero. Everything else remaining the same, we then expect the conservative technological policy $A = 0$ to be more probable.

3.3. Sustainable and optimal growth with $\gamma < 0$

As mentioned before, the HARA utility function is negative when $\gamma < 0$. In general, this does not prevent the existence of well-defined growth paths. The objective simply is to make discounted utility the least negative. An advantage of this case is that utility is bounded by zero and the optimal-growth problem, thus, is reminiscent of Ramsey's model. No strong assumption is necessary on the discount rate: optimal growth paths exist even with a zero discount rate (α_0 is strictly positive and (14) is well defined for $\delta = 0$).

The difficulty in the present framework is that optimization has bearing not only on the intertemporal allocation of consumption but also on the life expectancy of society. With negative instantaneous utility, it is quite clear that optimizing society's welfare cannot involve keeping the probability of survival equal to one. Quite the contrary, optimality may require reducing society's life expectancy.

Expression (15), furthermore, shows us that in the case of an irreversible technological choice, sustainable growth is necessarily suboptimal in the case where $\gamma \leq 0$. Because the RHS of (15) and the social welfare W^* are both negative for all values of A, the derivative of social welfare with respect to A, $(\partial W^*/\partial A)$, is positive. Thus, it is optimal to choose the largest possible value of A, instead of the sustainable growth solution $A = 0$. Optimality, here, requires us to choose the most productive technology $\bar{A}$, even though this may drastically reduce the probability of survival. In fact, this latter condition is good for society, since it reduces the expected time during which it will suffer with negative utility levels.

Of course, the same result applies to the reversible case.[16] It may easily be proven that the shadow price, ρ, of the hazard rate Λ in (27) now is positive: a higher hazard increases welfare by reducing the time period over which society suffers a negative utility. It follows that condition (20), which ensures that sustainable growth ($A = 0$) is optimal, cannot be satisfied.

We thus have the following proposition:

Proposition 3. *Under Assumption 1 and the HARA utility function such that $\gamma \leq 0$, sustainable growth cannot be optimal because instantaneous social utility is negative.*

To reverse the result of Proposition 3, while keeping the same elasticity of utility, we must simply perform a linear transformation of the HARA utility function—which makes its upper asymptotic bound positive instead of zero, as in the original specification (6). Such a transformation is equivalent to introducing the utility of basic survival of the human species:

$$u^0(c) = s + \frac{1-\gamma}{\gamma}\left(\frac{c - C}{1-\gamma}\right)^{\gamma} \quad \text{with } s > 0 \text{ and } \gamma \leq 0. \tag{28}$$

In the transformed HARA function, the constant s stands for the utility of society being alive at some point in time. It follows that $u^0(c)$ now varies monotonically over the interval $]-\infty, s[$, instead of $]-\infty, 0[$, when c runs over the interval $]C, \infty[$.

It is a simple matter to show that if s is large enough in (28), then sustainable growth again becomes locally optimal. The optimized welfare level W^* now is given by

$$W^* = \frac{s}{\delta + Am} + \frac{(1-\gamma)^{1-\gamma}}{\gamma}\alpha^{\gamma-1}\left[k_0 - \frac{C}{\mu + Ap}\right]^{\gamma} \tag{29}$$

and its derivative with respect to A is

$$\frac{\partial W^*}{\partial A} = -\frac{sm}{(\delta + Am)^2} - \frac{(1-\gamma)^{1-\gamma}}{\gamma}\left[k_0 - \frac{C}{\mu + Ap}\right]\alpha^{\gamma-2}(m - p\gamma) \\ \left[k_0 - \frac{C}{\mu + Ap}\right]^{\gamma-1}\frac{pC}{(\mu + Ap)^2}. \tag{30}$$

We then have

Proposition 4. *Under Assumption 1b, a strictly positive discount rate δ, a HARA modified function (28) such that $\gamma \leq 0$, and an irreversible technological choice, a necessary condition for sustainable growth to be optimal is*

$$\delta \leq \mu$$

and

$$\frac{sm}{\delta^2} > (1-\gamma)^{1-\gamma}\left(k_0 - \frac{C}{\mu}\right)^{\gamma-1}\left(\frac{\delta - \mu\gamma}{1-\gamma}\right)^{\gamma-2}\left[\left(\frac{\delta - \mu\gamma}{1-\gamma}\right)\frac{pC}{\mu^2} - \frac{m - p\gamma}{\gamma}\left(k_0 - \frac{C}{\mu}\right)\right] \tag{31}$$

Utility functions of the type (28) are quite general and have much to recommend them. Essentially, they imply an infinite instantaneous loss when consumption approaches its minimum survival level, whereas they place a (positive) upper bound on utility when consumption becomes increasingly large. Proposition 4 is then left to show that no general equivalence can exist between sustainable and optimal growth. Equivalence is possible only for some combinations of the parameters of the model. As could be expected, this equivalence is more likely when the utility of survival—or the upper bound of utility—s is large in comparison with other parameters. As before, it is also more likely when the risk element of technological innovation is large and its productivity effect is small—the RHS of (31) is a decreasing function of m and an increasing function of p.

A more detailed look at (31) may show that the RHS decreases from $+\infty$ to zero when the initial stock of capital, in excess of what is necessary to guarantee the minimum consumption level, goes from zero to infinity. Thus, sustainable growth is consistent with optimal growth only for affluent societies. Unlike the previous case ($\gamma \in]0, 1[$), however, no analytical expression exists of the threshold of $(k_0 - C/\mu)$ beyond which sustainable growth is optimal. It may also be seen in (31) that, unlike the previous case, it is no longer necessary for the minimum consumption level C to be strictly positive. For sustainable growth to be optimal, a minimum value on k_0 is required, even when $C = 0$.

4. Summary and conclusion

The paper began by asking what would be the ultimate implications of considering sustainable growth as the requirement that a society's consumption never decreases, even in the presence of a stochastic environment that, with no control, might in some instance lead to the extinction of the human species. We have shown that this problem could be made equivalent to a standard optimal-growth problem, where the two state variables were, on one hand, the stock of capital used in economic activity and, on the other hand, the probability of survival. Under certain conditions, the solution to that problem is a corner solution, which requires that the probability of survival be driven to one as fast as possible, at the cost of economic growth. However, these conditions depend importantly on the initial development level of the economy and the elasticity of utility with respect to consumption. Two economies possessing different initial stocks of capital or elasticities of utility will not follow the same development strategy. Therefore, sustainable growth's desirability depends on the initial development level and some social valuation of consumption. This is important because it shows that sustainable growth, as defined in the present paper, may ultimately have to do with a simple cardinalization problem of the social utility of consumption, rather than with noncardinal parameters such as risk aversion or intertemporal substitution.

It must be stressed that the comparison between sustainable and optimal growth is sometimes impossible because of the nonexistence of optimal-growth paths. This is the case, in particular, in which no discount rate, or too low a discount rate, exists in the social objective function. In such circumstances, the sustainable-growth concept has a clear autonomy with respect to the usual optimality criterion.

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Notes

1. A weaker form of that definition would be that the probability of a drop in consumption—and related variables—be as small as physically possible. Note that this definition is consistent also with the concept of *intergenerational altruism*, often referred to in connection with sustainability.
2. Indeed, the increasing risk of extinction is one justification for keeping intact the stock of natural capital. This is one of the so-called strong sustainability rules proposed by Pearce, Markandya, and Barbier (1989).
3. Note that with our specification, it is possible to interpret δ not as the usual discount rate, but as some kind of unavoidable hazard rate such that the probability of survival at time t is $e^{-\delta t}$. This interpretation of δ may be less debatable than discounting utilities because an increasing risk of extinction introduces an asymmetry between generations (see Parfit, 1984). However, by definition, growth cannot be sustainable with δ positive: the human species will disappear with probability one at an infinite horizon.
4. In Baranzini and Bourguignon (1994), we compare the results obtained in our simple model with those resulting from the canonical model of growth-cum-environment.
5. See, for instance, Bertola (1993).
6. Therefore, we discarded the familiar quadratic utility function. Also note that in our framework cardinality matters. See more on that subject below.
7. Indeed, these interpretations of γ are correct only asymptotically. For instance, instantaneous elasticity of utility is given by $\epsilon = \gamma c/(c - C)$.
8. The limit case $\gamma = 1$ represents a linear utility function and is uninteresting. It is well known that the other limit case, $\gamma = 0$, corresponds to the logarithmic utility function. Its properties are closer to the case discussed below.
9. Nuclear technology or fossil fuel burning may be considered as irreversible technological choices (at least regarding the consequences they engender).

10. The irreversible case can easily be solved with $b > 0$, but we use $b = 0$, keeping the model to its essential elements, thus simplifying intuitive interpretation of the results.
11. Even the overtaking criterion fails to work in this case. See on that point the short survey of issues linked to the value of the discount rate in optimal-growth models by Epstein (1987).
12. See Barro (1990) for an analogous condition. However, it is somewhat surprising that little attention is given to that condition in numerous recent endogenous-growth models based on a framework comparable to (5).
13. This argument is also in Rosen (1988).
14. A rigorous proof will require too much space, and thus we will not develop here our argument in more depth.
15. Equation (27) with $b = 0$ is stronger than (16) because $\gamma \in]0, 1[$, $\mu\gamma < \delta \leq u$ and thus $\alpha_0 < 1$.
16. We will not present this case in detail because it sheds no new light on our problem.

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