Haute école de gestion de Genève
CRAG - Centre de Recherche Appliquée en Gestion
Cahier de recherche

Liquidity Adjusted VaR Model: An Extension

Emmanuel Fragnière, Nils S. Tuchschmid and Qun Zhang

Cahier : N° HES-SO/HEG-GE/C--08/13/1--CH

2008

Liquidity Adjusted VaR Model: An Extension

Emmanuel Fragnière, Nils S. Tuchschmid and Qun Zhang

Cahier de recherche

Dezember 2008

Résumé

Cet article propose une nouvelle structure pour le calcul de la “Value at Risk” ajustée de la liquidité des marchés, appelée également LVaR. Le modèle présenté dans cet article est fondé sur une extension des travaux de Almgren et Chriss (1999 et 2000) qui portent sur des règles optimales de transaction dans un univers de sélection moyenne-variance. Au contraire du modèle d’Almgren et Chriss, nous exprimons ici la dynamique de prix sous la forme de taux de rendement et nous laissons les différentiels de prix entre l’achat et la vente et les impacts de prix dus aux transactions varier au cours du temps. Les résultats numériques montrent que notre formulation produit des résultats significativement différents de ceux d’Almgren et Chriss. Nous proposons également une formulation pour le calcul de la LVaR d’un portefeuille.

Mots-clés

Bid-ask spread, Liquidity adjusted value at risk (LVaR), Portefeuille LVaR; Price impact, Coût de transaction

Summary

This paper proposes a new framework for the calculation of liquidity adjusted value at risk, or LVaR. The model presented in this paper is extended from Almgren and Chriss’s mean-variance optimal trading approach (1999 and 2000). Contrary to Almgren and Chriss’s model, we express price fluctuation dynamic in terms of return and allow variations for both bid-ask spread and price impact. Numerical experiments show our formulation provides results significantly different from Almgren and Chriss’s solution. We then extend the model to calculate the LVaR for portfolios.

Keywords

Bid-ask spread, Liquidity adjusted value at risk (LVaR); Optimal trading solution, Portfolio LVaR; Price impact, Transaction cost

Liquidity Adjusted VaR Model: An Extension

Emmanuel Fragnière, Nils S. Tuchschmid and Qun Zhang*

Abstract

This paper proposes a new framework for the calculation of liquidity adjusted value at risk, or LVaR. The model presented in this paper is extended from Almgren and Chriss's mean-variance optimal trading approach (1999 and 2000). Contrary to Almgren and Chriss's model, we express price fluctuation dynamic in terms of return and allow variations for both bid-ask spread and price impact. Numerical experiments show our formulation provides results significantly different from Almgren and Chriss's solution. We then extend the model to calculate the LVaR for portfolios.

Key words: Bid-ask spread, Liquidity adjusted value at risk (LVaR); Optimal trading solution, Portfolio LVaR; Price impact, Transaction cost

Emmanuel Fragnière, CIA, Professor HEG and Lecturer at the University of Bath, Haute École de Gestion de Genève, Route de Drize 7, CH-1227 Carouge GE, Switzerland, Telephone: +41 (0) 22 3881724 Fax: +41 (0) 22 3881701, e-mail: Emmanuel.Fragniere@hesge.ch

Nils S. Tuchschmid, Professor HEG, Haute École de Gestion de Genève, Route de Drize 7, CH-1227 Carouge GE, Switzerland, Telephone: +41 (0) 22 3881700 Fax: +41 (0) 22 3881701, e-mail: Nils.Tuchschmid@hesge.ch

Qun Zhang, Researcher HEG, 73B/2 Kirk Brae, Edinburgh, United Kingdom, EH16 6JN, Telephone: +44 (0) 7912600501, e-mail: qzhang1985@hotmail.com

*Qun Zhang is the corresponding author.

Introduction

Developed over the last couple of decades, Value at risk (VaR) models have been widely used as main market risk management tool in the financial world. VaR estimates the likelihood of a portfolio loss caused by normal market movements over a given period of time. However, VaR fails to take into consideration the market liquidity impact. Its estimate is quite often based on mid-prices and it assumes that transactions do not affect market prices. Yet, large trading blocks impact prices and trading is costly. To overcome these problems, some researchers have proposed the calculation of liquidity adjusted VaR (LVaR). Different from the conventional VaR, LVaR takes both the size of the initial holding position and liquidity impact[1] into account. In this respect, LVaR can be seen as a complementary tool for risk managers in need of estimating market risk exposure and unwilling to disregard liquidity impact. Extended from Almgren and Chriss's optimal trading model (1999 and 2000), we here present a new framework for the calculation of liquidity adjusted VaR.

Bangia et al. (1999) proposed a practical but simple solution that is directly derived from the conventional VaR model in which liquidity factor is expressed as the bid-ask spread. While this approach avoids many complicated calculations, it fails to take into consideration endogenous liquidity factors. Hence, liquidity risk and LVaR are underestimated.

A more promising solution for LVaR estimation stems from the derivation of optimal trading strategies, as suggested by Almgren and Chriss (1999 and 2000). In their model, Almgren and Chriss adopted Holthausen, Leftwich, and Mayers's (1987) permanent and temporary market impact mechanisms, and assumed linear functions for both of them. By setting a sales completion period externally, they derived an optimal trading strategy defined as the strategy with the minimum variance of transaction cost, or of shortfall, for a given level of expected transaction cost[2]. With normality distribution, and mean and variance of transaction cost, LVaR can also be determined and minimized to derive optimal trading strategies. In this setting, LVaR is to be understood as the p-th percentile possible loss that a trading position can encounter when liquidity effects are incorporated into the risk measure computation. Later, Almgren (2003) extended this model by using a continuous-time approximation, and also introduced a non-linear and stochastic temporary market impact function. Another alternative is the liquidity discount approach presented by Jarrow and Subramanian (1997 and 2001). Similar to Almgren and Chriss's approach (1999 and 2000), it requires that the sales completion period be given as an exogenous factor. The optimal trading strategy is then derived by maximizing an investor's expected utility of consumption. Note that both approaches require externally setting a fixed horizon for liquidation. Aiming to overcome this problem, Hisata and Yamai (2000) extended Almgren and Chriss's approach by assuming a constant speed of sales and by using continuous approximation. They could derive a closed-form analytical solution for the optimal holding period. In this setting, the sales completion time thus becomes an endogenous variable. Yet, Hisata and Yamai's model relies on the strong assumption of a constant speed of sales.

For Krokhmal and Uryasev (2006), both Almgren and Chriss's or Jarrow and Subramanian's solutions are unable to dynamically respond to changes in market conditions. They therefore suggested a stochastic dynamic programming method and derived the optimal trading strategy by maximizing the expected stream of cash flow. Under their framework, the optimal trading strategy becomes highly dynamic, as it can respond to market conditions at each time step.

---

[1] Liquidity impact is commonly subcategorized into exogenous and endogenous liquidity factors. The former is normally measured by the bid-ask spread, and the latter is expressed as the price movement caused by market transactions (see Bangia 1999).

[2] Or inversely, a strategy that has the lowest level of expected transaction cost for a given level of variance.

However, stochastic programming makes their model quite complicated, which in turn dramatically lowers its potential for practical application. Another stochastic programming methodology for LVaR estimation can be found in Bertsimas and Lo (1998). Analytical expressions of the optimal execution strategies are derived by minimizing the expected trading cost over a fixed time horizon.

The methodology presented in this paper is extended from Almgren and Chriss's framework (1999 and 2000). Both exogenous and endogenous illiquidity factors are taken into account. The former is measured by the bid-ask spread, and the latter is expressed by linear market impact functions related to the quantity of sales. The model in this paper is built in a discrete-time manner[3], and the holding period is required to be determined externally. The permanent and temporary market impact mechanism proposed by Holthausen, Leftwich, and Mayers (1987) is adopted to formulate the market impact, and both permanent and temporary market impact are assumed as linear functions.

A methodology to calculate the portfolio LVaR is also proposed in this paper under the assumption that selling one specific security does not affect the prices of the other securities included in the portfolio. Finally, an approximation method is presented to enhance the computing efficiency of the portfolio LVaR model.

The paper is organized as follows. The single asset LVaR model is presented in Section two, and the corresponding case study and numerical examinations are performed in Section three. In Section four, the single asset LVaR model is extended to calculate the portfolio LVaR and an approximation method for portfolio LVaR calculation is proposed. Section five concludes and proposes future research directions.

2 LVaR Model

The model proposed in this paper is based on Almgren and Chriss's approach (1999 and 2000). While their model has shown to be an interesting methodology to calculate liquidity adjusted VaR, it also relies on a set of questionable hypotheses—arithmetic random walk for price fluctuation, constant bid-ask spread, and constant market impact coefficients. We here focus on relaxing these assumptions with the aim of improving accuracy of the LVaR calculation. First, we formulate the price fluctuation in terms of return. Second, we assume random variations of both bid-ask spread and market impact coefficients into the LVaR model.

We assume a trader sets his holding period T, externally. This holding period is itself divided into N intervals of equal length, $\tau = T/N$. The trading strategy is then defined as the quantity of shares sold in each time interval, that is denoted by $n_1,\ldots, n_k,\ldots,n_N$, where n_k is the number of shares that the trader plans to sell in the kth interval. Inversely, the quantity of shares that the trader plans to hold at time $t_k = k\tau$ is denoted by x_k. Suppose a trader has a position X (shares) that needs to be liquidated before time T, we thus have:

$$n_k = x_{k-1} - x_k$$

$$X = \sum_{k=1}^{N} n_k$$

[3] Although it is much easier to derive a closed-form analytical solution by using the continuous approximation (see Hisata and Yamai 2000, e.g.), the discrete-time model better fits reality, because a trader could not make sales in a continuous manner. In addition, a time interval τ might have to be long enough for the restoration of equilibrium. Continuous-time models cannot deal with that. In that respect, it becomes inconsistent with the assumption of the permanent and temporary market mechanism.

$$x_k = X - \sum_{j=1}^{k} n_j = \sum_{j=k+1}^{N} n_j, \quad k = 0,...,N.$$

Following Almgren and Chriss (1999 and 2000), we assume price fluctuation follows a discrete-time arithmetic random walk. Yet, we express it as a return generated process, as shown in equation [2-1] below. This new formulation helps circumvent some of the well known drawbacks of the arithmetic random walk process[4].

$$S_k = S_0\left(1+\mu_R\sum_{j=1}^{k}\tau + \sigma_R\sum_{j=1}^{k}\xi_j\sqrt{\tau}\right) - \sum_{j=1}^{k}\tau g\left(\frac{n_j}{\tau}\right) \qquad [2-1]$$

S_k is the equilibrium price after a sale, and S_0 is the initial price. μ_R and σ_R are the mean and standard deviation of the asset return series, respectively. The last term, $g(n_k/\tau)$, describes the permanent market impact from a sale.

However, it is assumed a trader cannot sell at this price. Due to a temporary market impact, the actual sale price is lower than this equilibrium price. Even though the formulation of actual sale prices is still debated[5], we here adopt Holthausen, Leftwich, and Mayers's (1987) framework with:

$$\tilde{S}_k = S_k - h\left(\frac{n_k}{\tau}\right) \qquad [2-2]$$

where $h(n_k/\tau)$ denotes the temporary market impact caused by selling n_k shares in interval τ. This temporary impact on price is assumed to be short-lived. This means it only exists temporarily after the trader makes the sale, and the equilibrium price, S_k, will be restored at or before the time t_k.

Linear functions are assumed for both temporary and permanent market impacts, as in Almgren and Chriss's framework (1999 and 2000). The permanent and temporary market impact functions are formulated as shown in equations [2-4] and [2-5].

$$g\left(\frac{n_k}{\tau}\right) = \gamma_k \cdot \frac{n_k}{\tau} = \frac{\gamma_k n_k}{\tau} \qquad [2-4]$$

$$h\left(\frac{n_k}{\tau}\right) = \varepsilon_k + \eta_k \cdot \frac{n_k}{\tau} = \varepsilon_k + \frac{\eta_k n_k}{\tau} \qquad [2-5]$$

where γ_k and η_k are the permanent and temporary market impact coefficients, respectively. ε_k denotes the cost of selling. It is estimated as half of the bid-ask spread in the *kth* interval, as we will assume no other transaction fees. We here assume market impact coefficients and cost of selling are random variables that follow arithmetic random walks. For γ_k, and η_k, we thus have:

$$\gamma_k = \gamma_0 + \sum_{j=1}^{k}\sigma_\gamma\xi_j\sqrt{\tau} \qquad [2-6]$$

[4] With this new formulation, asset prices do not take negative values, and price fluctuation becomes proportional to asset price. Therefore, we could expect this formulation to more accurately reflect the price dynamics.

[5] Almgren and Chriss adopted this framework, but they formulated the sale price in a different way as shown in equation [2-3].

$$\tilde{S}_k = S_{k-1} - h\left(\frac{n_k}{\tau}\right) \quad [2-3]$$

If the sale price is formulated using equation [2-3], there will be a problem. The sale price in the first time interval will be:

$$\tilde{S}_1 = S_o - h\left(\frac{n_1}{\tau}\right)$$

There is no standard deviation term in the price formulation for the first sale. It will cause a variance-missing problem. Therefore, equation [2-2] is adopted in this paper for the sale price formulation.

$$\eta_k = \eta_0 + \sum_{j=1}^{k} \sigma_\eta \xi_j \sqrt{\tau} \qquad [2-7]$$

γ_0 and η_0 are the initial value of permanent and temporary impact coefficients. σ_γ and σ_η are the corresponding standard deviations. ξ_j is a standard normally distributed random variable or Brownian motion.

For the formulation of ε_k, we employ a standardization process. Since bid-ask spreads tend to be proportional to asset prices, past observations may not accurately reflect current variations. Bangia et al. (1999) suggested the calculation of a relative bid-ask spread, which equals the bid-ask spread divided by the mid-price. By conducting this calculation, the bid-ask spread is expressed as a proportion of the asset price, thus the current bid-ask spread variation is sensitive to current asset price, rather than past observations. The relative bid-ask spread, as a normalizing device, can improve the accuracy of the bid-ask spread variation estimation. Following Bangia et al.'s suggestion, the half bid-ask spread in this paper is formulated as shown in equation [2-8],

$$\varepsilon_k = \frac{1}{2} S_0 \left(\bar{\varepsilon}_0 + \sum_{j=1}^{k} \sigma_{\bar{\varepsilon}} \xi_j \sqrt{\tau} \right) \qquad [2-8]$$

where $\bar{\varepsilon}_o$ and $\sigma_{\bar{\varepsilon}}$ are the initial value and standard deviation of the relative bid-ask spread, respectively. S_0 is the initial asset price. Consequently, the sale price in *kth* interval can be calculated as shown in equation [2-9].

$$\tilde{S}_k = S_0 \left(1 + \mu_R \sum_{j=1}^{k} \tau + \sigma_R \sum_{j=1}^{k} \xi_j \sqrt{\tau} \right) - \frac{1}{2} S_0 \left(\bar{\varepsilon}_0 + \sum_{j=1}^{k} \sigma_{\bar{\varepsilon}} \xi_j \sqrt{\tau} \right) - \gamma_0 \left(X - x_k \right)$$
$$- \sum_{j=1}^{k} \left(\sum_{m=1}^{j} \left(\sigma_\gamma \xi_m \sqrt{\tau} \right) n_j \right) - \frac{\left(\eta_0 + \sum_{j=1}^{k} \sigma_\eta \xi_j \sqrt{\tau} \right) n_k}{\tau} \qquad [2-9]$$

As we now have the sale price formulation in equation [2-9] for the *kth* interval, the total sale proceeds are naturally obtained by summing up this result over the whole N intervals, as shown in equation [2-10].

$$\sum_{k=1}^{N} n_k \tilde{S}_k = XS_0 + S_0 \sigma_R \sum_{k=1}^{N} x_{k-1} \xi_k \sqrt{\tau} + S_0 \mu_R \sum_{k=1}^{N} \tau x_{k-1} - \frac{1}{2} S_0 \bar{\varepsilon}_0 X - \frac{1}{2} S_0 \sigma_{\bar{\varepsilon}} \sum_{k=1}^{N} x_{k-1} \xi_k \sqrt{\tau} - \gamma_0 \sum_{k=1}^{N} n_k \left(X - x_{k-1} \right)$$
$$- \sum_{k=1}^{N} \sum_{j=1}^{k} \left(\sum_{m=1}^{j} \left(\sigma_\gamma \xi_m \sqrt{\tau} \right) n_j \right) n_k - \sum_{k=1}^{N} \frac{\left(\eta_0 + \sum_{j=1}^{k} \sigma_\eta \xi_j \sqrt{\tau} \right) n_k^2}{\tau}$$

$$= XS_0 + S_0 \sigma_R \sum_{k=1}^{N} x_{k-1} \xi_k \sqrt{\tau} + S_0 \mu_R \sum_{k=1}^{N} \tau x_{k-1} - \frac{1}{2} S_0 \bar{\varepsilon}_0 X - \frac{1}{2} S_0 \sigma_{\bar{\varepsilon}} \sum_{k=1}^{N} x_{k-1} \xi_k \sqrt{\tau} - \gamma_0 \sum_{k=1}^{N} n_k \left(X - x_{k-1} \right)$$
$$- \sum_{k=1}^{N} \left(\left(\sum_{j=1}^{k} \sigma_\gamma \xi_j \sqrt{\tau} \right) n_k \left(X - x_{k-1} \right) \right) - \sum_{k=1}^{N} \frac{\left(\eta_0 + \sum_{j=1}^{k} \sigma_\eta \xi_j \sqrt{\tau} \right) n_k^2}{\tau}$$

$$[2-10]$$

Consequently, transaction costs, or *TC*, can be derived by subtracting the total sale proceeds from the trader's initial holding value, that is:

$$
\begin{aligned}
TC &= XS_0 - \sum_{k=1}^{N} n_k \tilde{S}_k \\
&= -S_0\sigma_R \sum_{k=1}^{N} x_{k-1}\xi_k\sqrt{\tau} - S_0\mu_R\sum_{k=1}^{N}\tau x_{k-1} + \frac{1}{2}S_0\bar{\varepsilon}_0 X + \frac{1}{2}S_0\sigma_{\bar{\varepsilon}}\sum_{k=1}^{N} x_{k-1}\xi_k\sqrt{\tau} + \gamma_0\sum_{k=1}^{N} n_k\left(X - x_{k-1}\right) \\
&\quad + \sum_{k=1}^{N}\left(\left(\sum_{j=1}^{k}\sigma_\gamma\xi_j\sqrt{\tau}\right)n_k\left(X - x_{k-1}\right)\right) + \sum_{k=1}^{N}\frac{\left(\eta_0 + \sum_{j=1}^{k}\sigma_\eta\xi_j\sqrt{\tau}\right)n_k^2}{\tau}
\end{aligned}
\qquad [2-11]
$$

Recall that both ξ_k and ξ_j are assumed to follow standard normal distributions. Hence, their sum is also normally distributed and the mean and variance of transaction cost can easily be calculated, as shown below.

$$
E\left[TC\right] = -S_0\mu_R\sum_{k=1}^{N}\tau x_{k-1} + \frac{1}{2}S_0\bar{\varepsilon}_0 X + \gamma_0\sum_{k=1}^{N} n_k\left(X - x_{k-1}\right) + \eta_0\sum_{k=1}^{N}\frac{n_k^2}{\tau} \qquad [2-12]
$$

$$
V\left[TC\right] = \sum_{k=1}^{N}\left(\left(\sigma_R^2 + \frac{1}{4}\sigma_\varepsilon^2\right)S_0^2\tau x_{k-1}^2 + k\sigma_\gamma^2\tau\left(X - x_{k-1}\right)^2 n_k^2 + \frac{k\sigma_\eta^2 n_k^4}{\tau}\right) \qquad [2-13]
$$

Under this setting, the p-th percentile liquidity adjusted VaR can be formulated as:

$$
LVaR = E\left[TC\right] + \alpha_{cl}\sqrt{V\left[TC\right]} \qquad [2-14]
$$

where cl denotes the confidence level for the LVaR estimation, and α_{cl} is the corresponding percentile of the standard normal distribution. As expressed, LVaR measures a possible loss with a given position, taking into consideration both market risk conditions and liquidity effects.

The optimal trading strategy could be derived by minimizing the LVaR. The MP formulation of this optimization problem is thus written as:

$$
\begin{aligned}
\min_{n_k} \quad & E\left[TC\right] + \alpha_{cl}\sqrt{V\left[TC\right]} \\
s.t \quad & x_k = \sum_{j=k+1}^{N} n_j \\
& X = \sum_{k=1}^{N} n_k \\
& n_1, \ldots, n_N \geq 0.
\end{aligned}
$$

3 Case Study and Numerical Examinations

In this chapter, numerical examinations[6] are conducted for three different models:

- The first model takes Almgren and Chriss's original formulation[7] (1999 and 2000).
- The second model is based on our proposed extension but without incorporating randomness on both the bid-ask spread and the market impact coefficients.[8]

[6] The calculation of LVaR in this paper is conducted by using the *fmincon* function in the Matlab Optimization Toolbox.

[7] As explained in the previous section, Almgren and Chriss's model (1999 and 2000) has the missing-variance problem. This has to be corrected to run a proper numerical examination. The mean and variance of the transaction cost in this numerical examination is calculated by using the corrected equations shown below.

$$
E\left[TC\right] = \frac{1}{2}\gamma X^2 - \alpha\sum_{k=1}^{N}\tau x_{k-1} + \varepsilon X + \left(\frac{\eta}{\tau} - \frac{1}{2}\gamma\right)\sum_{k=1}^{N} n_k^2; \quad V\left[TC\right] = \sigma^2\sum_{k=1}^{N}\tau x_{k-1}^2
$$

[8] For the second model, the mean and variance of the transaction cost is calculated as shown below.

$$
E\left[TC\right] = -S_0\mu_R\sum_{k=1}^{N}\tau x_{k-1} + \frac{1}{2}\gamma X^2 + \varepsilon X + \left(\frac{\eta}{\tau} - \frac{1}{2}\gamma\right)\sum_{k=1}^{N} n_k^2; \quad V\left[TC\right] = \sigma_R^2 S_0^2\sum_{k=1}^{N}\tau x_{k-1}^2
$$

- The third and final model is based on the complete formulation with the incorporation of randomness on both the bid-ask spread and the market impact coefficients, as presented in the previous section.

We used data on JP Morgan's stock to conduct the numerical examinations[9]. Comparison between Almgren and Chriss's original formulation (model one) and our model two, in which we exclude randomness on bid-ask spread and liquidity market coefficients, is aimed at showing the effect of different price dynamics on LVaR estimation. The impact of incorporating randomness on liquidity factors (the bid-ask spread and the market impact coefficients) can then be seen between the results of model two and model three.

The holding period, T, is set to be 5 days, and we selected the time interval to be 0.5 day[10]. Thus, the total number of sales, N, is 10. Five different initial holdings are chosen for the numerical examinations, with the aim to observe how the initial position affects the LVaR calculation. A conventional VaR[11] is also calculated. The confidence level is set to be 95%. The results are summarized in the table 3-1 below.

Table 3-1: LVaR Calculation Results Summary

Initial Holding (shares)		10,000,000	5,000,000	1,000,000	500,000	100,000
Model one: Almgren and Chriss's original LVaR formulation	Total LVaR	9.237E+07	3.897E+07	5.963E+06	2.800E+06	5.247E+05
	LVaR per share	9.24	7.79	5.96	5.60	5.25
	LVaR Ratio	24.49%	20.66%	15.81%	14.85%	13.91%
Model two: Extended LVaR formulation without spread and market impact uncertainties	Total LVaR	2.775E+07	1.029E+07	1.283E+06	5.540E+05	8.941E+04
	LVaR per share	2.775	2.058	1.283	1.108	0.894
	LVaR Ratio	7.36%	5.46%	3.40%	2.94%	2.37%
Model three: Extended LVaR formulation with spread and market impact uncertainties	Total LVaR	3.031E+07	1.070E+07	1.310E+06	5.636E+05	8.987E+04
	LVaR per share	3.031	2.139	1.310	1.127	0.899
	LVaR Ratio	8.04%	5.67%	3.47%	2.99%	2.38%
Conventional VaR calculation	VaR per share	0.78	0.78	0.78	0.78	0.78
	VaR Ratio	2.07%	2.07%	2.07%	2.07%	2.07%

*LVaR per share = LVaR/Initial holding; LVaR Ratio = LVaR per share/Initial price.[12]

As expected, the numerical results show that LVaR estimates increase when initial holdings increase. This is true for all three LVaR formulations. Stated otherwise, the larger the initial holding, the stronger the market impact will be when a trader liquidates his position. As a corollary, the higher will be the liquidity adjusted Value-at-Risk. This is clearly a characteristic that distinguishes LVaR from traditional VaR.

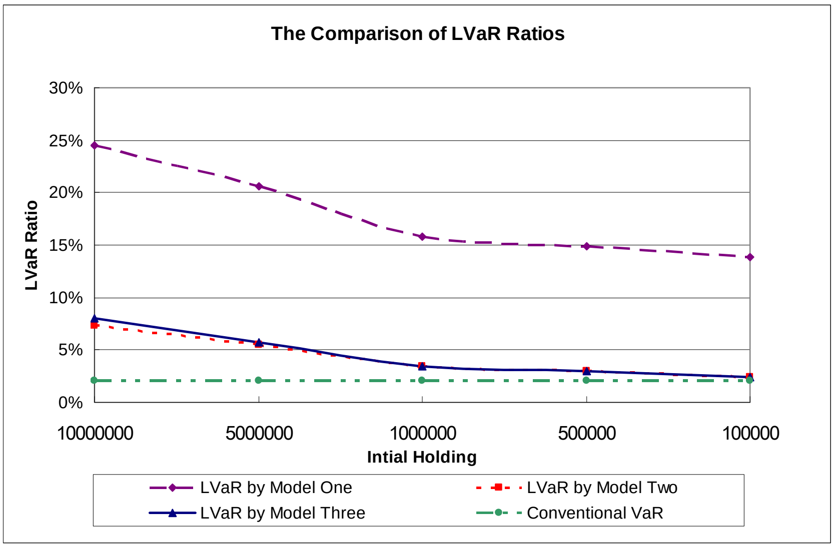
Comparison between LVaRs of model one (Almgren and Chriss's original formulation) and LVaRs of model two show that differences are significant. With an initial holding of 1,000,000 shares for instance, Almgren and Chriss's LVaR estimate is 5.963E+06. It corresponds to an LVaR ratio of 15.81%. The figure seems abnormally high compared to a

[9] Descriptive statistics are given in Appendix 3.
[10] The selection of holding period and time interval is arbitrary. The value of LVaR is directly proportional to the length of time interval, and inversely proportional to the holding period.
[11] Conventional VaR is calculated by using formula: $VaR = S_0(-\mu + \alpha_{cl}\sigma)$. In practice, VaR is commonly calculated by using geometric returns and the assumption of lognormal distribution. Since the LVaR models presented in this paper are all based on the assumption of arithmetic random walk and normal distribution, in order to make the comparison consistent, this VaR calculation formula is adopted. In addition, since the time interval is set to be 0.5, daily VaR has been adjusted by a factor of $\sqrt{0.5}$ to make consistent comparisons.
[12] The formulation of LVaR ratio is to show percentage of loss caused by sales. It makes the comparison easier.

traditional VaR of 2.07%. Contrarily, LVaR of model two drops to 1.283E+06, with a corresponding LVaR ratio of 3.40%., and offers a result that seems more realistic. The same conclusion is reached if one looks at the results of experiments when the initial holding is small. With 100,000 shares, the difference between LVaR of model two and VaR amounts to only 0.31%. The same is not true with the Almgren and Chriss's LVaR formulation. Its ratio is much higher than the corresponding VaR ratio, with a multiple bigger than 6 (a 2.07% ratio for a traditional VaR versus a 13.91% ratio for the Almgren and Chriss's LVaR). Yet, one would expect LVaR to be close to traditional VaR when holding is low. One should recall both LVaR models rest on the same formulation of market impact function. The only difference is found in the way price dynamics are formulated. Almgren and Chriss's high LVaRs are due to the somehow "high" standard deviation of JPM's daily price.[13]. However, high standard deviation seems inevitable if price fluctuation is formulated by a simple arithmetic random walk. In this respect, the price dynamic formulation we have adopted appears to be better suited to estimate LVaR, as the numerical results lie in a more reasonable range and are close to traditional VaRs when initial holdings get small.

LVaR estimates of model three are slightly higher than ones generated by model two. The reason is obvious. By including randomness on both the bid-ask spread and the market impact coefficients, the variance of transactions cost, or $V[TC]$, is enlarged. In turn, it causes an increase of the LVaR estimates. Of course, the bigger the initial holding is, the larger is the effect. When the trader is assumed to hold only 100,000 shares, LVaR estimates of models two and three are almost identical. When initial holding increases to 10,000,000 shares, the difference amounts to 0.68%.

Model comparisons can also be seen quite clearly from the figure below.



From a practical risk management point of view, this first set of results could lead one to believe that differences between our LVaR new formulation and a traditional VaR remains small, as long as the initial holding is not too big (less than 1,000,000 shares). Yet, it should be understood as a special case that stems from the value of the coefficients. In the table below for instance, both the initial values and standard deviations of the relative bid-ask spread and the permanent and temporary impact coefficients have been doubled[14].

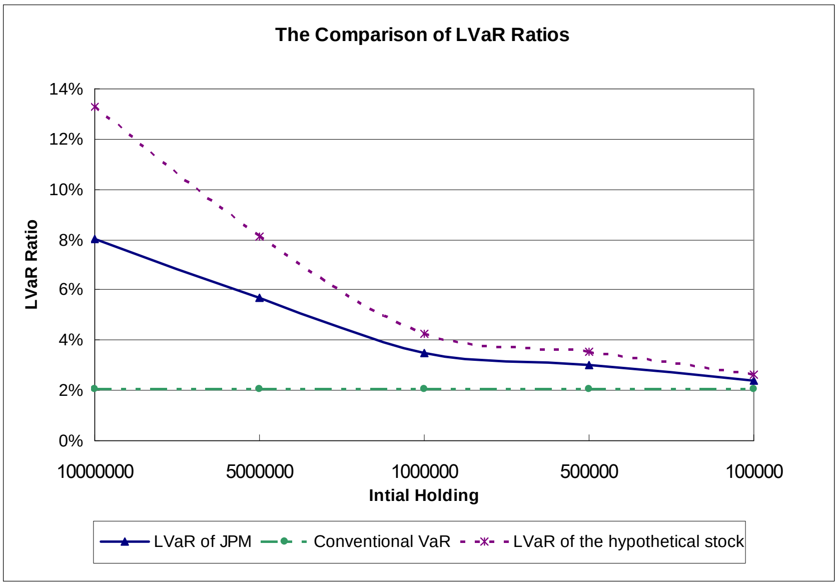
[13] From the table in Appendix 3, it can be seen that the standard deviation is 4.4037. There are 700 observations, which contain almost three years of daily prices. In such a long time period, it is not surprising to see such a high standard deviation. Nevertheless, it is doubtful whether the true daily price fluctuation could be so high.

[14] Details of data inputs are given in Appendix 4

Table 3-2: LVaR Calculation Results of the Hypothetical Stock

Initial Holding	10,000,000	5,000,000	1,000,000	500,000	100,000
Total LVaR	5.011E+07	1.528E+07	1.596E+06	6.679E+05	9.958E+04
LVaR per share	5.011	3.057	1.596	1.336	0.996
LVaR Ratio	13.28%	8.10%	4.23%	3.54%	2.64%
VaR Ratio	2.07%	2.07%	2.07%	2.07%	2.07%

Results in Table 3-2 show that LVaR and VaR can differ significantly, even if the initial holding is small. Comparisons can be seen more clearly from the figure below, which shows LVaR ratios of both the JPM stock and the hypothetical stock and their corresponding VaR ratios[15].



4 LVaR Estimation of Portfolio

To calculate a portfolio LVaR, we must deal with the correlation effect between each asset in the portfolio. Here we adopt Hisata and Yamai's assumption (2000), and make the hypothesis that the sale of one security does not affect the prices of the other securities in the portfolio. That is to say, the correlation only exists between price fundamentals, not between changes in prices that are only caused by transactions or selling pressure. Without this assumption, the LVaR calculation will then become extremely complicated. Finally, since the focus here is on portfolio LVaR, we assume the bid-ask spread and market impact coefficients to be constant to simplify the calculation.

It is supposed that there are *m* stocks in the portfolio. The variance covariance matrix of arithmetic returns is expressed as:

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \cdots & \sigma_{1m}^2 \\ \vdots & \ddots & \vdots \\ \sigma_{m1}^2 & \cdots & \sigma_m^2 \end{pmatrix}$$

σ_i^2 denotes the variance of the stock i's return, and σ_{ij}^2 denotes the covariance of the stock i and j's returns. Applying Cholesky's decomposition on this variance covariance matrix, the lower triangular matrix A can be calculated as:

[15] Since the hypothetical stock has the same price and return data inputs with JPM. Their VaR ratios are the same.

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mm} \end{pmatrix} \text{ (For } j > i,\ a_{ij} = 0\text{. And } \Sigma = AA^T\text{)}$$

We again fix the holding period of the portfolio to be *T*. Assuming the same return generating process as before, the actual sale price of stock *i* at the *kth* time interval can be expressed as:

$$\tilde{S}_{i,k} = S_{i,0}\left(1 + \mu_i \sum_{j=1}^{k} \tau + \sum_{j=1}^{k} \left(a_{i,1} \cdots a_{i,m}\right) \begin{pmatrix} \xi_{1,j} \\ \vdots \\ \xi_{m,j} \end{pmatrix} \sqrt{\tau}\right) - \varepsilon_i - \frac{\eta_i n_{i,k}}{\tau} - \gamma_i \left(X_i - x_{i,k}\right) \qquad [4-1]$$

where $\xi_{1,j} \cdots \xi_{m,j}$ are mutually independent standard Brownian motions with mean zero and variance one. Thus, the total sales value of the whole portfolio can be calculated as shown below.

$$\sum_{i=1}^{m}\sum_{k=0}^{N} n_{i,k}\tilde{S}_{i,k} = \sum_{i=1}^{m}\left(X_i S_{i,0} + S_{i,0}\mu_i \sum_{k=1}^{N} \tau x_{i,k-1} + S_{i,0}\sum_{k=1}^{N} x_{i,k-1}\left(a_{i,1} \cdots a_{i,m}\right)\begin{pmatrix} \xi_{1,k} \\ \vdots \\ \xi_{m,k} \end{pmatrix}\sqrt{\tau} - \varepsilon_i X_i - \sum_{k=1}^{N}\frac{\eta_i n_{i,k}^2}{\tau} - \gamma_i \sum_{k=1}^{N} n_{i,k}\left(X_i - x_{i,k}\right)\right)$$

$$[4-2]$$

Subtracting equation [4-2] from the initial value of the portfolio, the transaction cost is thus equal to:

$$\begin{aligned} TC &= \sum_{i=1}^{m} X_i S_{i,0} - \sum_{i=1}^{m}\sum_{k=0}^{N} n_{i,k}\tilde{S}_{i,k} \\ &= \sum_{i=1}^{m}\left(-S_{i,0}\mu_i \sum_{k=1}^{N} \tau x_{i,k-1} - S_{i,0}\sum_{k=1}^{N} x_{i,k-1}\left(a_{i,1} \cdots a_{i,m}\right)\begin{pmatrix} \xi_{1,k} \\ \vdots \\ \xi_{m,k} \end{pmatrix}\sqrt{\tau} + \varepsilon_i X_i + \sum_{k=1}^{N}\frac{\eta_i n_{i,k}^2}{\tau} + \gamma_i \sum_{k=1}^{N} n_{i,k}\left(X_i - x_{i,k}\right)\right) \\ &= \sum_{i=1}^{m}\left(-S_{i,0}\mu_i \sum_{k=1}^{N} \tau x_{i,k-1} - S_{i,0}\sum_{k=1}^{N} x_{i,k-1}\left(a_{i,1} \cdots a_{i,m}\right)\begin{pmatrix} \xi_{1,k} \\ \vdots \\ \xi_{m,k} \end{pmatrix}\sqrt{\tau} + \varepsilon_i X_i + \frac{1}{2}\gamma_i X_i^2 + \left(\frac{\eta_i}{\tau} - \frac{1}{2}\gamma_i\right)\sum_{k=1}^{N} n_{i,k}^2\right) \end{aligned}$$

$$[4-3]$$

From equation [4-3], the mean and variance of the transaction cost of this portfolio are easily calculated as:

$$E[TC] = \sum_{i=1}^{m}\left(\frac{1}{2}\gamma_i X_i^2 - S_{i,0}\mu_i \sum_{k=1}^{N} \tau x_{i,k-1} + \varepsilon_i X_i + \left(\frac{\eta_i}{\tau} - \frac{1}{2}\gamma_i\right)\sum_{k=1}^{N} n_{i,k}^2\right) \qquad [4-4]$$

$$\begin{aligned} V[TC] &= V\left[\sum_{i=1}^{m} S_{i,0}\sum_{k=1}^{N} x_{i,k-1}\left(a_{i,1} \cdots a_{i,m}\right)\begin{pmatrix} \xi_{1,k} \\ \vdots \\ \xi_{m,k} \end{pmatrix}\sqrt{\tau}\right] \\ &= V\left[\sum_{i=1}^{m} S_{i,0}\left(a_{i,1} \cdots a_{i,m}\right)\begin{pmatrix} \xi_{1,k} \\ \vdots \\ \xi_{m,k} \end{pmatrix}\sqrt{\tau}\sum_{k=1}^{N} x_{i,k-1}\right] \\ &= \sum_{k=1}^{N}\begin{pmatrix} S_{1,0}x_{1,k-1} \\ \vdots \\ S_{m,0}x_{m,k-1} \end{pmatrix}^T \sum \tau \begin{pmatrix} S_{1,0}x_{1,k-1} \\ \vdots \\ S_{m,0}x_{m,k-1} \end{pmatrix} \qquad [4-5] \end{aligned}$$

Finally, the liquidity adjusted VaR can be formulated in the same way as shown in equation [2-14], and the MP formulation of this optimization problem then becomes:

$$\min_{n_{i,k}} \quad E[TC]+\alpha_{cl}\sqrt{V[TC]}$$

$$s.t \quad x_{i,k}=\sum_{j=k+1}^{N} n_{i,j}$$

$$X_i=\sum_{k=1}^{N} n_{i,k}$$

$$n_{i,k}\geq 0.$$

where the optimal trading strategy is no longer a vector but a $m\times N$ matrix as show below.

$$Optimal\,Trading\,Strategy=\begin{pmatrix} n_{1,1} & \cdots & n_{1,N} \\ \vdots & \ddots & \vdots \\ n_{m,1} & \cdots & n_{m,N} \end{pmatrix}$$

As long as the number of stocks in the portfolio is small, the optimal solution could be easily generated with a high computing efficiency. However, in practice, a portfolio could contain many assets. The number of variables in the objective function (that equals $m\times N$) can then become quite large. In that case, generating an optimal solution quickly gets difficult, and the computing efficiency is low[16]. To overcome this obstacle and make the methodology applicable in practice, we suggest the following approximation method and test for its degree of accuracy.

It is assumed that the optimal trading strategy derived by the portfolio LVaR model does not diverge much from the optimal derivation in the single asset LVaR model. If this assumption holds, the following approximation procedure could be done:

- First, the single asset LVaR model is adopted to generate optimal trading strategy for each asset in the portfolio.
- Second, the mean and variance of the transaction cost is calculated by substituting the optimal trading strategies into equations [4-4] and [4-5].
- Third, the approximate LVaR of the portfolio could be calculated by substituting the mean and variance calculated in the first two procedures into the equation [2-14].

Needless to mention, the assumption is strong. The correlation between stock returns should influence the optimal trading strategy. However, as long as this approximation method does not show a significant error, it could be regarded as accurate enough to substitute the portfolio LVaR calculated by the proper model presented above.

To test this assumption, we performed a simple numerical examination by limiting the number of assets in the portfolio. We then estimated the portfolio LVaR generated by both the proper method and the approximated method. Finally, we compared the results by changing the correlation coefficients between assets with the aim to examine the sensitivity of the error term.

Suppose a portfolio consists of two stocks—JP Morgan and Citigroup (descriptive statistics are given in Appendix 1)—with initial holdings of 10,000,000 and 20,000,000 shares respectively[17]. Once again, we fix the holding period to be 5 days, and the length of time

[16] The calculation of LVaR in this paper is conducted by using Matlab Optimization Toolbox. When the number of variables is large, it may reach the maximum number of iterations allowed in the estimation process, and make the convergence more difficult.
[17] The initial holdings that we choose for all the numerical examinations in this section are purposely very large. With significant holding, errors caused by approximation method should be more easily detectable.

interval to 0.5 day. Inputting these data into the model, portfolio LVaRs are calculated by changing the correlation coefficient from 1 to -1. Numerical results are shown in the table 4-1.

Table 4-1: Portfolio LVaR calculated by proper model vs. Approximated Portfolio LVaR

Correlation Coeff.	LVaR (proper model)	Ratio	LVaR (approx. model)	Ratio	Ratios Diff.
1	75,459,398	10.01%	75,459,930	10.01%	0.00%
0.75	73,547,572	9.75%	73,551,650	9.75%	0.00%
0.5	71,482,803	9.48%	71,502,059	9.48%	0.00%
0.25	69,224,803	9.18%	69,274,169	9.19%	0.01%
0	66,711,747	8.85%	66,811,330	8.86%	0.01%
-0.25	63,839,596	8.46%	64,018,490	8.49%	0.02%
-0.5	60,405,609	8.01%	60,711,331	8.05%	0.04%
-0.75	55,887,254	7.41%	56,419,623	7.48%	0.07%
-1	45,373,871	6.02%	47,582,770	6.31%	0.29%

Recall that for the approximation method to be applied, the optimal trading strategy first needs to be run for each single asset. These results are shown in Table 4-2.

Table 4-2: Optimal Trading Strategies Generated by single asset LVaR model

Time Period	1	2	3	4	5
JPM	1,513,574	1,336,118	1,186,567	1,062,120	960,327
Citi group	2,542,370	2,367,389	2,214,889	2,083,498	1,972,006
Time Period	**6**	**7**	**8**	**9**	**10**
JPM	879,098	816,700	771,754	743,242	730,499
Citi group	1,879,366	1,804,691	1,747,257	1,706,503	1,682,030

Based on these optimal trading strategies, the approximated portfolio LVaR could then be calculated by following the approximation procedure as presented above. Results are shown in the fourth and fifth columns of Table 4-1.

Interestingly enough, we first note that the portfolio LVaR calculated by both the proper model and the approximation method increased with the correlation coefficient. Inversely, the LVaR estimates are the lowest when the correlation is -1. This fits with general portfolio theory. We can then note that the two methods of estimation do not show sizeable differences. As expected, the approximate portfolio LVaR is higher than the portfolio LVaR calculated by the proper model, as the former fails to incorporate any diversification effect into the optimal trading strategy. Yet, results show that differences are almost negligible when the correlation coefficient is positive and close to one. As the correlation coefficient turns out to be negative, the error term increases. This is because the optimal trading strategies for a single asset and a portfolio tend to diverge more and more as the correlation coefficient decreases. However, it clearly remains within an acceptable range. The maximum difference amounts to 0.29% only while both LVaRs are exceeding 6%.

To further test the accuracy of the approximation method, we conducted another experiment by adding two stocks to the portfolio, UBS and Bank of America (BoA), with initial holdings of 10,000,000 for both (Data inputs can be seen in Appendix 4). Other conditions are the same. Finally, we set six arbitrary correlation coefficient matrixes[18] shown in Table 4-3.

[18] In order to better detect the accuracy of the approximation method, we choose extreme values for the correlation coefficient matrixes to conduct the examination.

Table 4-3: Correlation Coefficient Matrixes

Index 1	JPM	Citigroup	UBSN	BoA	Index 2	JPM	Citigroup	UBSN	BoA
JPM	1	1	1	1	**JPM**	1	0	0	0
Citigroup	1	1	1	1	**Citigroup**	0	1	0	0
UBSN	1	1	1	1	**UBSN**	0	0	1	0
BoA	1	1	1	1	**BoA**	0	0	0	1
Index 3	**JPM**	**Citigroup**	**UBSN**	**BoA**	**Index 4**	**JPM**	**Citigroup**	**UBSN**	**BoA**
JPM	1	-1	-1	-1	**JPM**	1	-1	1	-1
Citigroup	-1	1	1	1	**Citigroup**	-1	1	-1	1
UBSN	-1	1	1	1	**UBSN**	1	-1	1	-1
BoA	-1	1	1	1	**BoA**	-1	1	-1	1
Index 5	**JPM**	**Citigroup**	**UBSN**	**BoA**	**Index 6**	**JPM**	**Citigroup**	**UBSN**	**BoA**
JPM	1	-1	0	-1	**JPM**	1	1	-1	0
Citigroup	-1	1	0	1	**Citigroup**	1	1	-1	0
UBSN	0	0	1	0	**UBSN**	-1	-1	1	0
BoA	-1	1	0	1	**BoA**	0	0	0	1

Numerical results are presented in Table 4-4, and the optimal trading strategy generated by the single asset LVaR model for each of the four stocks is shown in Appendix 5.

Table 4-4: Portfolio LVaR calculated by proper model vs. Approximated Portfolio LVaR

Correlation Coeff. Matrix Index	LVaR (proper model)	Ratio	LVaR (approx. model)	Ratio	Ratios Diff.
1	81675107	3.71%	81755935	3.71%	0.00%
2	59171763	2.69%	59759692	2.71%	0.03%
3	58449533	2.65%	61755801	2.80%	0.15%
4	42060797	1.91%	45658858	2.07%	0.16%
5	53526271	2.43%	55360480	2.51%	0.08%
6	42263030	1.92%	44587919	2.02%	0.11%

Results are close to the ones previously obtained. When assets' prices are positively correlated, errors are almost negligible. When correlation coefficients are negative, the errors tend to increase, yet remain marginal. In our examples, the maximum is reached at the index 4 matrix with an LVaR ratio difference of 0.16%. The four assets portfolio LVaR is 1.91%. Its approximation yields 2.07%. Clearly, even if it deserves further analysis and research, the suggested approximation method shows a high potential for practical application.

5 Conclusion

This paper presents a simple but practical framework for the calculation of liquidity adjusted value at risk (LVaR). Relaxing some restrictive assumptions of Almgren and Chriss (1999 and 2000), we propose a robust way to estimate possible losses stemming from exiting positions when both adverse market conditions and liquidity effects are taken into consideration. Overall, our numerical results indicate that our model significantly improves the estimation of LVaR. We also developed the portfolio LVaR model and suggested an approximation technique that has a clear, practical application.

Yet, LVaR models still remain in their infancy, calling for further research that we envision going into three main directions.

First is the estimation of market resiliency. If tightness and market depth can be captured, market resiliency remains a concept more difficult to grasp, and therefore more difficult to estimate. However, resiliency is an important factor for liquidity risk modeling, and especially for LVaR estimation. Indeed, it directly relates to the choice of the length of time intervals between each sale. The model presented in this paper is a discrete-time model. It requires the time interval to be determined externally. A short time interval could reduce LVaR. However, a short interval may not be able to guarantee restoration of equilibrium. On the contrary, a long time interval could guarantee that the price reaches equilibrium, yet, LVaR will be enlarged. Therefore, to make an accurate estimation for LVaR, it is important to choose a proper length for the sale interval, which highlights the significance of the market resiliency estimation.

Second is the formulation of market impact functions. How to formulate the market impact functions more accurately is not a new issue. It heavily affects the accuracy of the whole LVaR estimation. Nevertheless, it is the weakest or most questionable part of the whole LVaR calculation. In this paper, we have assumed that functions are linear, which is commonly adopted in most research. Yet, if there are more accurate methods to estimate the market impact functions (e.g. a power law function to formulate nonlinear market impact functions as in Jarrow and Subramanian (2001) and Almgren (2003)) there is a need to find the right balance between accuracy and complexity.

Third and finally, further research is needed as far as portfolio LVaR calculation is concerned. The portfolio LVaR model presented in this paper could only be conducted with high efficiency when the number of assets was relatively small. The approximation method we propose greatly simplifies LVaR computation, and its results indicate that errors remain negligible. However, approximation errors increase when risky assets are negatively correlated. Further research should thus be conducted and other methods could be proposed and tested. Along this line, backtested results should help when analyzing the accuracy of the solution we proposed as well. Indeed, using intraday market prices and measuring the realized sale proceeds of trading strategies will put ex-ante LVaR into real market conditions and allow for better assessment of the accuracy of its estimates.

Last, but not least, it remains worthwhile concluding that LVaR, as traditional VaR, remains a relevant risk metric under normal market conditions. No accurate LVaR could be estimated if markets crunch and liquidity completely dries up.

Appendix

Appendix 1 Data Inputs for Numerical Examination of Single Asset LVaR Model

JP Morgan's daily stock data from September 13, 2005 to June 24, 2008 are chosen to perform the numerical examinations. There are 700 observations. For the estimation of temporary and permanent market impact coefficients, Almgren and Chriss didn't propose a specific method. They assumed that:

· For the temporary market impact, trading each 1% of the daily volume incurs price depression of one bid-ask spread.

· For the permanent market impact, trading 10% of the daily volume will cause a significant influence to the price, and incur price depression of one bid-ask spread.

Numerical examinations conducted in this chapter follow these assumptions for simplicity[19]. Therefore, the temporary and permanent market impact coefficients are calculated as shown in equations [A1] and [A2], respectively.

$$temporary\ impact\ coefficient = \frac{bid\ ask\ spread}{1\% \times volume} \qquad [A1]$$

$$permanent\ impact\ coefficient = \frac{bid\ ask\ spread}{10\% \times volume} \qquad [A2]$$

The details of data inputs are summarized in the table below.

The Summary of Data Input

Asset Price	Initial value	37.72
	Standard Dev	4.4037
Return	Mean value	3.015E-04
	Standard Dev	1.796E-02
Bid-ask Spread	Initial value	0.05
	Standard Dev	0.0381
Relative Bid-ask Spread	Initial value	1.326E-03
	Standard Dev	8.430E-04
Permanent Impact Coefficient	Initial value	5.3443E-08
	Standard Dev	5.5987E-08
Temporary Impact Coefficient	Initial value	5.3443E-07
	Standard Dev	5.5987E-07
Drift		0.0051

19 The estimation of permanent and temporary market impact coefficients is a complicated issue. The specific methodology can be found in Holthausen, Leftwich, and Mayers's paper, "The Effect of Large Block Transactions on Security Prices: A Cross-Sectional Analysis" (1987) and "Large-Block Transactions, the Speed of Response, and Temporary and Permanent Stock-Price Effects" (1990). The estimation requires separated seller-initiated data and buyer-initiated data. Due to the limited access to the data source, the accurate estimation could not be performed in this paper. Since this paper focuses on the LVaR modeling, not the estimation of market impact coefficients, Almgren and Chriss's simple assumption is adopted to conduct all the numerical examinations in this paper.

Appendix 2 Data Inputs of the Hypothetical Stock

The Summary of Data Input

Asset Price	Initial value	37.72
	Standard Dev	4.4037
Return	Mean value	3.015E-04
	Standard Dev	1.796E-02
Relative Bid-ask Spread	Initial value	2.652E-03
	Standard Dev	8.430E-04
Permanent Impact Coefficient	Initial value	1.07E-07
	Standard Dev	1.12E-07
Temporary Impact Coefficient	Initial value	1.07E-06
	Standard Dev	1.12E-06

Appendix 3 Data Inputs for the Examination of the Portfolio LVaR Model (Two assets)

The data for JP Morgan and Citigroup were collected from September 13, 2005 to June 24, 2008. There are 700 observations each. The details of the data inputs are summarized in the table below.

The Summary of Data Input

Asset Price	Initial value	37.72	18.85
	Standard Dev	4.4037	10.3909
Return	Mean value	3.015E-04	-1.063E-03
	Standard Dev	1.796E-02	1.923E-02
Bid-ask Spread	Initial value	0.05	0.07
	Standard Dev	0.0381	0.0347
Permanent Impact Coefficient	Initial value	5.3443E-08	3.0466E-08
	Standard Dev	5.5987E-08	2.0689E-08
Temporary Impact Coefficient	Initial value	5.3443E-07	3.0466E-07
	Standard Dev	5.5987E-07	2.0689E-07

Appendix 4 Data Inputs for the Examination of the Portfolio LVaR Model (Four Assets)

Stock		**JPM**	**Citigroup**	**UBSN**	**BAC**
Asset Price	Initial value	47.66	50.8	67.035	54.85
	Standard Dev	3.9496	1.6927	5.8325	3.6973
Return	Mean value	1.1696E-03	4.3297E-04	1.2232E-03	8.7458E-04
	Standard Dev	1.0457E-02	8.3561E-03	1.3462E-02	8.2245E-03
Bid-ask Spread	Initial value	0.04	0.03	0.05	0.04
	Standard Dev	0.0134	0.0140	0.1261	0.0172
Relative Bid-ask Spread	Initial value	8.3928E-04	5.9055E-04	7.4588E-04	7.2926E-04
	Standard Dev	3.2083E-04	2.9261E-04	2.1714E-03	3.6209E-04
Permanent Impact Coefficient	Initial value	2.0708E-08	1.7445E-08	6.5757E-08	4.7983E-08
	Standard Dev	2.0677E-08	1.8821E-08	2.9793E-07	2.0953E-08
Temporary Impact Coefficient	Initial value	2.0708E-07	1.7445E-07	6.5757E-07	4.7983E-07
	Standard Dev	2.0677E-07	1.8821E-07	2.9793E-06	2.0953E-07

Appendix 5 Optimal Trading Strategies Generated by Single Asset LVaR Model

Period	**JPM**	**Citi**	**UBSN**	**BoA**
1	1726490	1770824	1558344	1366762
2	1409624	1435472	1270043	1226811
3	1196624	1213566	1120914	1119264
4	1040399	1050586	1023122	1034888
5	921990	925579	951359	968308
6	832481	829008	895105	916199
7	766987	755772	848996	876368
8	722394	702737	809946	847251
9	696347	667691	776058	827638
10	686665	648765	746112	816510

Bibliography

Almgren, R. and Chriss, N. (1999), Value under liquidation. *Risk*, 12 (December), pp. 61-63.
Almgren, R. and Chriss, N. (2000), Optimal execution of portfolio transactions. *Journal of Risk*, 3(Winter), pp.5-39.
Almgren, R. (2003), Optimal Execution with Nonlinear Impact Functions and Trading-Enhanced Risk. *Applied Mathematical Finance*, 10, pp.1-50.
Bangia, A., Diebold, F., Schuermann, T. and Stroughair, J. (1999), Liquidity on the outside. *Risk*, 12 (June), pp. 68-73.
Bertsimas, D and Lo., A. (1998), Optimal control of execution costs. *Journal of Financial Markets*, 1, pp. 1-50.
Fernandez, F. A., (1999), Liquidity Risk: New Approaches to Measurement and Monitoring. Securities Industry Association, *Working Paper*.
Hisata, Y. and Yamai, Y. (2000), Research toward the practical application of liquidity risk evaluation methods, *Monetary and Economic Studies*, (December, 2000), pp. 83-128.
Holthausen, R. W., Leftwich, R. W., and Mayers, D. (1987), The Effect of Large Block Transactions on Security Prices: A Cross-Sectional Analysis. *Journal of Financial Economics*, 19, 1987, pp. 237-268.
Holthausen, R. W., Leftwich, R. W., and Mayers, D. (1990), Large-Block Transactions, the Speed of Response, and Temporary and Permanent Stock-Price Effects. *Journal of Financial Economics*, 26, 1990, pp. 71-95.
Jarrow, R. A. and Subramanian, A. (1997), Mopping up liquidity. *Risk*, 10 (December), pp.170-173.
Jarrow, R. A. and Subramanian, A. (2001), The liquidity discount, Mathematical Finance, Volume 11, Number 4, pp. 447-474.
Krokhmal P. and S. Uryasev, (2006) A Sample-Path Approach to Optimal Position Liquidation, Annals of Operations Research, Published Online, November, pp. 1-33

Cahiers de recherche du Centre de Recherche Appliquée en Gestion (CRAG) de la Haute Ecole de Gestion - Genève

CRAG – Centre de Recherche Appliquée en Gestion
Haute école de gestion - Genève
Campus de Battelle, Bâtiment F
7, route de Drize – 1227 Carouge – Suisse
✉ crag@hesge.ch
www.hesge.ch/heg/crag
✆ +41 22 388 18 18
🖷 +41 22 388 17 40

2006

- N° HES-SO/HEG-GE/C--06/1/1--CH
  Andrea BARANZINI
  Damien ROCHETTE
  "La demande de récréation pour un parc naturel. Une application au Bois de Pfyn-Finges, Suisse"

- N° HES-SO/HEG-GE/C--06/2/1--CH
  Giovanni FERRO LUZZI
  Yves FLÜCKIGER
  Sylvain WEBER
  "A Cluster Analysis of Multidimentional Poverty in Switzerland"

- N° HES-SO/HEG-GE/C--06/3/1--CH
  Giovanni FERRO LUZZI
  Sylvain WEBER
  "Measuring the Performance of Microfinance Institutions"

- N° HES-SO/HEG-GE/C--06/4/1--CH
  Jennifer D'URSO
  "L'eau de boisson : Aspects logistiques et attitude du consommateur"

- N° HES-SO/HEG-GE/C--06/5/1--CH
  Jennifer D'URSO
  "La gestion publique de l'eau en Suisse"

- N° HES-SO/HEG-GE/C--06/6/1--CH
  Philippe THALMANN
  Andrea BARANZINI
  "Gradual Introduction of Coercive Instruments in Climate Policy"

- N° HES-SO/HEG-GE/C--06/7/1--CH
  Andrea BARANZINI
  Caroline SCHAERER
  José RAMIREZ
  Philippe THALMANN
  “Feel it or Measure it. Perceived vs. Measured Noise in Hedonic Models”

- N° HES-SO/HEG-GE/C--06/8/1--CH
  José RAMIREZ
  Anatoli VASSILIEV
  “An Efficiency Comparison of Regional Employment Offices Operating under Different Exogenous Conditions”

- N° HES-SO/HEG-GE/C--06/9/1--CH
  José RAMIREZ
  Joseph DEUTSCH
  Yves FLÜCKIGER
  Jacques SILBER
  “Export Activity and Wage Dispersion : The Case of Swiss Firms”

- N° HES-SO/HEG-GE/C--06/10/1--CH
  Joëlle DEBELY
  Gaëtan DERACHE
  Emmanuel FRAGNIERE
  Jean TUBEROSA
  “Rapport d’enquête : sondage Infobésité”

- N° HES-SO/HEG-GE/C--06/11/1--CH
  Andrea BARANZINI
  José RAMIREZ
  Cristian UGARTE ROMERO
  “Les déterminants du choix de (dé)localisation des entreprises en Suisse”

- N° HES-SO/HEG-GE/C--06/12/1--CH
  Catherine EQUEY BALZLI
  Jean TUBEROSA
  David MARADAN
  Marie-Eve ZUFFEREY BERSIER
  “Étude du comportement des PME/PMI suisses en matière d’adoption de système de gestion intégré. Entre méconnaissance et satisfaction.”

- N° HES-SO/HEG-GE/C--06/13/1--CH
  Joëlle DEBELY
  Magali DUBOSSON
  Emmanuel FRAGNIÈRE
  “The pricing of the knowledge-based services : Insight from the environmental sciences”

2007

- N° HES-SO/HEG-GE/C--07/1/1--CH
  Andrea BARANZINI
  Caroline SCHAERER
  “A Sight for Sore Eyes
  Assessing the value of view and landscape use on the housing market”

- N° HES-SO/HEG-GE/C--07/2/1--CH
  Joëlle DEBELY
  Magali DUBOSSON
  Emmanuel FRAGNIÈRE
  “The Travel Agent: Delivering More Value by Becoming an Operational Risk Manager”

- N° HES-SO/HEG-GE/C--07/3/1--CH
  Joëlle DEBELY
  Magali DUBOSSON
  Emmanuel FRAGNIÈRE
  “The Consequences of Information Overload in Knowledge Based Service Economies”

- N° HES-SO/HEG-GE/C--07/4/1--CH
  Lucie BEGIN
  Jacqueline DESCHAMPS
  Hélène MADINIER
  “Une approche interdisciplinaire de l’intelligence économique”

- N° HES-SO/HEG-GE/C--07/5/1--CH
  Journée de la recherche HEG 2007
  “Recueil des communications”

- N° HES-SO/HEG-GE/C--07/6/1--CH
  Sylvain WEBER
  Andrea BARANZINI
  Emmanuel FRAGNIÈRE
  “Consumers Choices among Alternative Electricity Programs in Geneva – An Empirical Analysis”

2008

- N° HES-SO/HEG-GE/C--08/1/1--CH
  Andrea BARANZINI
  José RAMIREZ
  Sylvain WEBER
  “The Demand for Football in Switzerland : An Empirical Estimation”

- N° HES-SO/HEG-GE/C--08/2/1--CH
  Giuseppe CATENAZZO
  Gaëtan DERACHE
  Emmanuel FRAGNIÈRE
  Patricia HUGENTOBLER
  Jean TUBEROSA
  "Rapport d'enquête préliminaire : Dessine-moi un service ! Entreprises et administration : comment concevoir et valoriser vos services"

- N° HES-SO/HEG-GE/C--08/3/1--CH
  Nguyen VI CAO
  Emmanuel FRAGNIÈRE
  Jacques-Antoine GAUTHIER
  Marlène SAPIN
  Eric WIDMER
  "Optimizing the marriage market through the reallocation of partners : An application of the linear assignment model"

- N° HES-SO/HEG-GE/C--08/4/1--CH
  Magali DUBOSSON
  Emmanuel FRAGNIÈRE
  Bernard MILLIET
  "A Control System Designed to Address the Intangible Nature of Service Risks"

- N° HES-SO/HEG-GE/C--08/5/1--CH
  Giuseppe CATENAZZO
  Jennifer D'URSO
  Emmanuel FRAGNIÈRE
  Jean TUBEROSA
  "Influences of Public Ecological Awareness and Price on Potable Water Consumption in the Geneva Area"

- N° HES-SO/HEG-GE/C--08/6/1--CH
  Alexandra BROILLET
  Magali DUBOSSON
  "Analyzing Web 2.0 Internet users in order to drive innovation in distribution strategy of luxury watches : A netnography analysis"

- N° HES-SO/HEG-GE/C--08/7/2--CH
  Alexandra BROILLET
  Magali DUBOSSON
  "Luxury e-services at the pre- and after-sales stages of the decision making process: Watch, car, art and travel blogs analysis"

- N° HES-SO/HEG-GE/C--08/8/1--CH
  Nicolas BUGNON
  René SCHEIDER
  "OPACs et utilisateurs - L'étude ACUEIL démontre les comportements de recherche et propose des outils simplifiés et flexibles"

- N° HES-SO/HEG-GE/C--08/9/1--CH
  Giuseppe CATENAZZO
  Emmanuel FRAGNIERE
  "Attitudes Regarding New Enterprise Risk and Control Regulations by the Active Population of the Geneva Area"

- N° HES-SO/HEG-GE/C--08/10/1--CH
  Giuseppe CATENAZZO
  Emmanuel FRAGNIERE
  "Identifying Bank Runs Signals through Sociological Factors: An Empirical Research in the Geneva Area"

- N° HES-SO/HEG-GE/C--08/11/1--CH
  Caroline SCHAERER
  Andrea BARANZINI
  "Where and How Do Swiss and Foreigners Live? Segregation in the Geneva and Zurich Housing Markets"

- N° HES-SO/HEG-GE/C--08/12/1--CH
  Giuseppe CATENAZZO
  Jennifer D'URSO
  Emmanuel FRAGNIERE
  "Elements of perception regarding sustainable development in Geneva"