

Dynamic Cognitive-Social Particle Swarm Optimization

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Abstract—Particle Swarm Optimization (PSO) is a heuristic optimization algorithm based on the modeling of the behavior of fishes and birds flock. This paper proposes a better version of PSO, named Dynamic Cognitive-Social PSO “DCS-PSO”, for global minima search by introducing optimal and dynamic cognitive and social scaling parameters without taking into consideration the inertia term. Furthermore, the velocity of each particle is controlled systematically at each iteration to avoid local minimum traps and to converge quickly and reliably towards the global minimum. The proposed algorithm is more suitable for high dimensional optimization problems and it has gotten over the limitations of classical Particle Swarm Optimization. Several experiments have been carried out, using the proposed DCS-PSO, to optimize thirteen benchmark functions and an important improvement has been observed, not only in terms of reaching the best global solutions but also in terms of convergence speed, compared to the existing benchmark approaches.

Keywords—dynamic parameters, particle swarm optimization, velocity, convergence

I. INTRODUCTION

The Particle Swarm Optimization (PSO) is an intelligent algorithm first introduced by [1]. PSO is inspired by the social collective behavior of fishes (or birds) flocking. Imagine the following scheme: a group of fishes searches randomly for food in an area in which there is only one piece. The fishes know that food is far away on every move but Not all know where food is. The effective strategy for finding food is certainly to follow the fish nearest to the food.

PSO is a heuristic global optimization method (also an algorithm), which learned from this scheme. Each potential solution in the search space is a “fish” called “particle”. Each particle learns from its own experience and from other particles during the repeated flights. The learning from own experience may be denoted as *cognitive learning*. In this case, the particle memories the *personal best* position i.e. the best solution visited so far by itself. The learning from others may be denoted as *social learning* and the particle memories in this case the best solution visited by any particle of the swarm which called as *global best* position [2].

The PSO is an iterative algorithm, initializing with a set of random solutions, called a swarm of particles, and then searches for optima by updating iterations. Each particle could be a potential solution and it has several parameters such as the

velocity, the best position visited by the particle so far, and the current position [3]. In a d -dimensional search space, the position and the velocity of each particle is represented by d -dimensional vectors, which guide the moving of the particles according to the individual and the collective knowledge. The velocity and position of the i^{th} particle at iteration t are represented as follows:

$$v_i(t) = (v_{i1}, v_{i2}, \dots, v_{id}) \quad (1)$$

$$x_i(t) = (x_{i1}, x_{i2}, \dots, x_{id}) \quad (2)$$

In the original version of PSO, the velocity and position, at every iteration t , are updated as follows:

$$v_i(t+1) = v_i(t) + r_1 c_1 (p_i(t) - x_i(t)) + r_2 c_2 (p_g(t) - x_i(t)) \quad (3)$$

$$x_i(t+1) = v_i(t+1) + x_i(t) \quad (4)$$

where, $v_i(t)$ is the velocity of i^{th} particle at iteration t , $p_i(t)$ refers to the *personal best* position of the i^{th} particle, $p_g(t)$ refers to the *global best* position of the entire swarm, r_1 and r_2 are two random values in the interval $[0,1]$ and the two acceleration constants c_1 and c_2 are known respectively as *cognitive* and *social* scaling parameters. They are generally fixed as 2.0. However, [4] in their study, c_1 and c_2 are function in terms of iteration t as follows:

$$c_1 = c_2 = 2 + \frac{t_{max}-t}{5t_{max}}, \quad (5)$$

where t_{max} represents the maximum number of iterations.

Since the introduction of the standard PSO, several modifications are introduced and other versions have been proposed. [5, 6], because the basic PSO [1] has its limitations (e.g. in the field of plasmonics [7]). The idea of regulating of the velocity has been presented in [8]. The authors study the effects of velocity limitations and use different maximum values to controlling the velocity.

To regulate the velocity, most common methods consist in multiplying by factors when updating the value of velocity.

The key input in this research is the integration of dynamic factors to regulate the velocity at each iteration, thus for two reasons: One is to avoid local minimum traps and to converge quickly and reliably towards the global minimum whereas the other one is to achieve a balance between exploitation and exploration. The proposed algorithm achieves better results

than the standard PSO algorithm and it is competitive when compared with other benchmark methods.

II. PARTICLE SWARM OPTIMIZATION VERSIONS

The success of an optimization algorithm depends generally on bringing about a compromise between global search and local search throughout the whole search space. Considering of this, an *inertia weight*, w , of a particle is brought into the original equation and the velocity is updated as follow:

$$v_i(t+1) = wv_i(t) + c_1r_1(p_i(t) - x_i(t)) + c_2r_2(p_g(t) - x_i(t)) \quad (6)$$

According to [9], most of the common PSO algorithms developed until now include the *inertia weight* coefficient, and it is commonly mentioned as the Standard PSO (SPSO). Table I summaries several existing *inertia weight* strategies reported in the literature.

Reference [20] proposes the introduction of a *constriction factor* χ to insure the convergence of the PSO. In this case, the velocity is updated according to the following equation:

$$v_i(t+1) = \chi[v_i(t) + \varphi_1r_1(p_i(t) - x_i(t)) + \varphi_2r_2(p_g(t) - x_i(t))] \quad (7)$$

where χ is given by:

$$\chi = \frac{2}{|2 - \varphi - \sqrt{\varphi^2 - 4\varphi}|} \quad (8)$$

and $\varphi = \varphi_1 + \varphi_2$, $\varphi > 4$.

Thus, the equation 7 can be written as:

$$v_i(t+1) = \chi v_i(t) + \chi \varphi_1 r_1 (p_i(t) - x_i(t)) + \chi \varphi_2 r_2 (p_g(t) - x_i(t)) \quad (9)$$

We can note that equation 9 becomes analogous to equation 6 with:

$$w = \chi, c_1 = \chi \varphi_1 \text{ and } c_2 = \chi \varphi_2 \quad (10)$$

A comparison between the *constriction factor* and the *inertia weight* is proposed in the study of [12]. It has been concluded that the use of the constriction factor gives better quality solutions. When this factor is used, generally a value of 4.1 for φ is taken, and thus $\chi \approx 0.7298$, with: $\varphi_1 = \varphi_2 = 2.05$ and $c_1 = c_2 = 1.49445$. Note that χ can be taken as a function. However, [21] proposed a *constriction factor* based on the cosine defined in the following equation:

$$\chi = \frac{\cos\left(2\frac{\pi}{t_{max}}(t - t_{max})\right) + 2.428571}{4} \quad (11)$$

where t_{max} is the maximum iteration number.

TABLE I. DIFFERENT INERTIA WEIGHT STRATEGIES

N.	Inertia weight formula	Related description	Reference
1	$w = \text{constant}$	Constant <i>inertia weight</i>	[10]
2	$w = 0.5 + \frac{\text{rnd}()}{2}$	Random w	[11]
3	$w = w_{max} - \frac{w_{max} - w_{min}}{t_{max}} \cdot t$	w_{max}, w_{min} : initial and final values of w	[12]
4	$w = 1.1 + \frac{p_g}{\text{Avg}(p_i)}$	t_{max} : maximum number of iterations	[13]
5	$w = w_{min} + (w_{max} - w_{min}) \left(\frac{t_{max} - t}{t_{max}}\right)^n$	p : global best position p_i : personal best position	[14]
6	$w = w_{min} - (w_{max} - w_{min}) \cdot \lambda^{(t-1)}$ $\lambda = 0.95$	Simulated annealing inertia weight	[15]
7	$w = w_{min} - (w_{max} - w_{min}) \cdot \frac{\text{rank}_i}{\text{total population}}$	Rank_i : position of the i^{th} particle	[16]
8	$w = w_{min} \cdot u^{t_{\text{term}}}$	u : constant value in the range [1.0001, 1.005]	[17]
9	$w = w_{max} \cdot Z + (w_{min} - w_{max}) \cdot \frac{t_{max} - t}{t_{max}}$	Z : chaotic term.	[18]
10	$w(t+1) = w_i(t) - (w_i(t) - 0.4) \cdot \exp\left(-\left (p_g - p_i) \cdot \frac{t}{t_{max}}\right \right)$	Adaptive nonlinear inertia weight	[19]

III. THE PROPOSED METHOD

The proposed method in this paper, named Dynamic Cognitive-Social PSO “DCS-PSO”, is characterized by the non-use of the *inertia* term and thus focusing on the *social* and *cognitive* parts of the PSO along with controlling the speed values. The idea of this strategy is based on the fact that the particles in the swarm should follow those which have shown an improvement in their respective position in the local search space. So, a leaping strategy is used in order to escape converging prematurely towards a local minimum. A new *cognitive* and *social* functions $c_1(t)$ and $c_2(t)$ are introduced. The function $c_1(t)$ is incorporated using a logarithm function defined in equation (12). The position of each particle is

limited by the range of $[x_{min}, x_{max}]$, where x_{min} and x_{max} are respectively the lower and the upper bounds of variables.

$$c_1(t) = a(t)^{\alpha(x_{max} - x_{min})} \cdot \log(\|p_i(t) - x_i(t)\| + \epsilon) \quad (12)$$

where, ϵ is a very small positive value, $a(t)$ is a *dynamic cognitive* parameter and $\alpha(x_{max} - x_{min})$ is a *dynamic regulator exponent* defined as follow:

$$\alpha(x_{max} - x_{min}) = \begin{cases} 5, & |x_{max} - x_{min}| \leq 10 \\ 1, & |x_{max} - x_{min}| > 10 \end{cases} \quad (13)$$

Four initial values of $a(0)$ are tested: $(\chi\pi)^{\frac{1}{4}}$, $\cos(\sqrt{\chi\pi})$, $\sin(\sqrt{\chi\pi})$ and e , while the *social* function is given as $c_2(t) = (\chi\pi)^\alpha$.

Every time, when a particle is stuck at a local minimum, the velocity is near to zero, the particle could make a lead using the logarithm function and thus skipping a possible local optimum. To allow larger leaps at the start of iterations to explore the whole search space and small leaps for a refinement in specific solution-space regions, the idea of linear decreasing *cognitive* parameter, with a dynamic damping coefficient $d(t)$ and a damping coefficient k , is used as follow:

$$a(t+1) = d(t) \cdot a(t) \quad (14)$$

$$d(t+1) = k \cdot d(t) \quad (15)$$

where the initial values of $d(0)$ and k are taken respectively 0.9995 and 0.99999, while $a(0)$ will take the four different values given above.

The range of the velocity $[v_{min}, v_{max}]$ is selected to be:

$$\begin{cases} v_{max} = 0.9[x_{max} - x_{min}] \\ v_{min} = -0.1[x_{max} - x_{min}] \end{cases} \quad (16)$$

So, the velocity equation of the proposed DCS-PSO will be updated as:

$$v_i(t+1) = \begin{cases} v_{max} & \text{if } h(t) > v_{max} \\ v_{min} & \text{if } h(t) < v_{min} \\ h(t) & \text{otherwise} \end{cases} \quad (17)$$

where

$$h(t) = r_1 c_1(t)(p_i(t) - x_i(t)) + r_2 c_2(t)(p_g(t) - x_i(t))$$

The strategy of the proposed DCS-PSO is described in algorithm 1.

ALGORITHM 1. PSEUDOCODE OF DCS-PSO ALGORITHM

1. **Initialization**
 - Step 1:* Set the initial parameters
 - Step 2:* Initialize randomly the velocity and the position of the particles
 - Step 3:* Set the actual position of particles equal to the first best position values
 - Step 4:* Evaluate the fitness values of particles and determine the global best position.
 2. **Update of velocity and position of particles**
 - Step 5:* Update the velocity of particles by equation (16)
 - Step 6:* Apply velocity limits
 - Step 7:* Update the position of particles by equation (4)
 - Step 8:* Apply position limits
 3. **Update of personal and global best positions**
 - Step 9:* Update best and global positions
 - Step 10:* Update dynamic parameter and coefficients
 4. **Convergence procedure**
 - Step 11:* Repeat step 5 to 11 until convergence or until reacting the maximum number of iterations.
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IV. RESULTS

A. Comparison of DCS-PSO with PSO variants

In this section, DCS-PSO is compared with existing variants of PSO [22,23] on 8 numerical functions taken from the report CEC2005 [24]. Table II gives a detailed description of the 8 benchmark functions including the search spaces and the global minimums. The dimension of all those functions are fixed to 30. The swarm population size, the maximum number of iterations, and the independent runs are respectively chosen as 30, 10000 and 20. The PSO algorithm will stop when the maximum number of iterations is reached. For the comparison to be judicious, we consider, like in the study of [22], that results below 10^{-30} are assumed to be 0.

The data presented in Table III, with the exception of the DCS-PSO algorithm, are taken directly from the literature [22]. The comparison is based on mean fitness values, where, it can be seen that DCS-PSO has a high-quality solution in the search space compared to other PSO variants. The results in terms of precision of the solution are clearly better. It can be noted that the results for the four different initial values of $a(t)$ tested are almost close. However, the use of $a(0) = e$ remains very interesting and gives the best result.

B. Comparison of DCS-PSO with other optimization algorithms

To test the performance of the proposed method, other seven common benchmark functions are tested [25]. Table II summaries the description of these functions. From Table IV, it can be observed that the proposed algorithm has performed significantly better over the standard PSO for all functions f_9 - f_{15} . For functions f_9 et f_{14} , the results are comparable with these of DE [28] and P-PSO [27]. The result obtained for f_{12} are better over P-PSO [27] and w-PSO [27]. The results for f_{10} , f_{11} , f_{13} and f_{15} were compared only with the standard PSO due to the lack of results in the literature.

The results given in Table IV demonstrates that the DCS-PSO algorithm has achieved better results and are usually very close to the global minimums of the functions tested. This is certainly due, on the one hand, to the strategy of leaping which allow to not falling into the local minima (see Fig. 1). Indeed, if a particle found stuck i.e. its personal best position and its current position have the same values; what does mean that its velocity is close to zero. In this case, the logarithmic function allows the particle to do a jumping (take a big value) and thus escape converging towards a local minimum. Note that equation (17) ensures that the value of velocity always remains admissible. On the other hand, to the advantage of the cognitive and damping parameters to achieve on a compromise between the capacity of local and global search by improving the power of PSO to overcome of the local extremums with fast convergence speed near to the global value. To verify the fast convergence, DCS-PSO is compared with the standard PSO (Fig. 2). From this figure, we can note a great advantage of DCS-PSO in offering a faster convergence and we can observe that few iterations are needed to reach best solutions, regardless the dimension [29].

TABLE II. DIFFERENT BENCHMARK FUNCTIONS USED [24, 25]

Name of function	Equation	Range Space	$f(x^*)$	Features
Sphere	$f_1(x) = \sum_{i=1}^n x_i^2$	$[-100,100]$	0	Multimodal, scalable, separable, differentiable
Rosenbrock	$f_2(x) = \sum_{i=1}^n (b(x_{i+1} - x_i^2) + (x_i - a)^2)$, $b = 100$, $a = 1$	$[-2.048,2.048]$	0	Unimodal, scalable, non-separable, differentiable
Schwefel	$f_3(x) = 418.9829 \cdot n - \sum_{i=1}^n x_i \sin(\sqrt{x_i})$	$[-500,500]$	0	Unimodal, scalable, partially-separable, differentiable
Weierstrass	$f_4(x) = \sum_{i=1}^n (\sum_{k=0}^{k_{max}} [a^k \cos(2\pi b^k (x_i + 0.5))]) - n \sum_{k=0}^{k_{max}} [a^k \cos(2\pi b^k 0.5)]$ $a = 0.5, b = 3, k_{max} = 20$	$[-0.5,0.5]$	0	Multimodal, scalable, separable, differentiable
Shifted Schwefel 1.2	$f_5(x) = \sum_{i=1}^n (\sum_{j=1}^i z_j)^2 - 450$	$[-100,100]$	-450	Shifted, unimodal, scalable, non-separable
Shifted Rastrigin	$f_6(x) = \sum_{i=1}^n (z_i^2 - 10 \cos(2\pi z_i) + 10) - 330$	$[-5,5]$	-330	Shifted, multimodal, scalable, separable
Shifted Rotated Rastrigin	$f_7(x) = \sum_{i=1}^n (z_i^2 - 10 \cos(2\pi z_i) + 10) - 330$	$[-5,5]$	-330	Shifted, rotated, multimodal, scalable, non-separable
Shifted Rotated Weierstrass	$f_8(x) = \sum_{i=1}^n (\sum_{k=0}^{k_{max}} [a^k \cos(2\pi b^k (z_i + 0.5))]) - n \sum_{k=0}^{k_{max}} [a^k \cos(2\pi b^k 0.5)]$ $a = 0.5, b = 3, k_{max} = 20$	$[-0.5,0.5]$	0	Shifted, rotated, multimodal, scalable, non-separable
Ackley	$f_9(x) = -20 \exp\left(-0.2 \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2}\right) - \exp\left(\frac{1}{n} \sum_{i=1}^n \cos 2\pi x_i\right) + 20 + e$	$[-32,32]$	0	Multimodal, non-convex, differentiable
Schwefel 2.20	$f_{10}(x) = \sum_{i=1}^n x_i $	$[-100,100]$	0	Multimodal, non-convex, differentiable
Periodic	$f_{11}(x) = 1 + \sum_{i=1}^n \sin(x)^2 - 0.1 e^{\sum_{i=1}^n x_i^2}$	$[-10,10]$	0	Multimodal, non-separable, differentiable, non-convex
SumSquare	$f_{12}(x) = \sum_{i=1}^n i x_i^2$	$[-10,10]$	0	Unimodal, convex
Alpine	$f_{13}(x) = \sum_{i=1}^n x_i \sin(x_i) + 0.1 x_i $	$[-10,10]$	0	Multimodal, non-convex, differentiable, non-separable,
Griewank	$f_{14}(x) = \frac{1}{4000} \sum_{i=1}^n x_i^2 - \prod_{i=1}^n \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	$[-600,600]$	0	Unimodal, non-convex
Exponential	$f_{15}(x) = -\exp(-0.5 \sum_{i=1}^n x_i^2)$	$[-1,1]$	-1	Unimodal, convex, differentiable, non-separable

$z = x - o$ for f_5 and f_6 and $z = (x - o) \cdot M$ for f_7 and f_8 . $o = (o_1, o_2, \dots, o_n)$ is the (shifted) global optimum and M is the orthogonal square matrix.

TABLE III. DCS-PSO PERFORMANCE COMPARISON WITH THE 9 PSO VARIANTS

Function	ScPSO	CLPSO	UPSO	SG-PSO	SP-PSO	Local PSO-cf	Global PSO-cf	Local PSO-w	Global PSO-w	Proposed method: DCS-PSO (a)			
										$a = (\chi\pi)^{\frac{1}{4}}$	$a = \cos(\sqrt{\chi\pi})$	$a = \sin(\sqrt{\chi\pi})$	$a = e$
f_1	0.00e+00	0.00e+00	9.07e+03	0.00e+00	1.48e-06	1.34e+04	1.00e+03	1.79e+02	1.00e+03	0.00e+00	0.00e+00	0.00e+00	0.00e+00
f_2	3.10e+01	4.81e+01	1.03e+03	2.08e+02	2.20e+01	1.72e+03	1.07e+02	1.27e+02	1.07e+02	0.00e+00	8.11e+01	3.23e+00	0.00e+00
f_3	2.21e+03	5.46e+03	7.98e+03	4.20e+03	4.03e+03	7.36e+03	4.46e+03	6.41e+03	4.46e+03	3.81e-04	4.73e+03	3.81e-04	3.81e-04
f_4	3.30e-01	2.42e-01	2.64e+01	1.60e+00	0.00e+00	2.70e+01	1.83e+00	4.37e+00	1.83e+00	0.00e+00	0.00e+00	0.00e+00	0.00e+00
f_5	3.97e+04	5.49e+06	7.85e+06	5.70e+06	1.73e+05	1.01e+07	6.00e+06	5.89e+06	6.00e+06	1.29e+05	2.29e+05	1.65e+05	1.60e+04
f_6	3.75e+01	2.16e+02	2.48e+02	1.57e+02	1.29e+02	4.01e+02	1.38e+02	2.93e+02	1.38e+02	5.21e+00	7.04e+00	5.52e+00	4.63e+00
f_7	1.74e+02	1.70e+02	3.90e+02	2.31e+02	1.72e+02	5.72e+02	2.28e+02	3.83e+02	2.28e+02	-3.07e+01	-2.86e+01	-3.78e+01	-5.25e+01
f_8	2.73e+01	3.27e+01	3.98e+01	2.77e+01	2.89e+01	3.65e+01	2.99e+01	3.37e+01	2.99e+01	4.01e+01	4.00e+01	3.94e+01	3.26e+01

TABLE IV. DCS-PSO PERFORMANCE EVALUATION

Function	PSO	GA [26]	BitABC [26]	P-PSO [27]	w-PSO [27]	DE [28]	FEP [28]	Proposed method: DCS-PSO (a)			
								$a = (\chi\pi)^{\frac{1}{4}}$	$a = \cos(\sqrt{\chi\pi})$	$a = \sin(\sqrt{\chi\pi})$	$a = e$
f_9	1.63e+01	1.22e-02	1.22e-04	4.22e-15	1.05e-06	9.70e-08	1.80e-02	4.99e-14	7.08e-01	5.00e-14	8.88e-16
f_{10}	4.58e+02	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	3.82e-13	4.99e+00	3.26e-13	0.00e+00

f_{11}	1.75e+00	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	0.90e+00	0.90e+00	0.90e+00	0.90e+00
f_{12}	1.87e+03	N.A.	N.A.	7.12e+01	1.59e-12	N.A.	N.A.	0.00e+00	5.00e+00	0.00e+00	0.00e+00
f_{13}	1.44e+01	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	9.63e-11	7.01e-07	2.86e-11	4.27e-02
f_{14}	1.37e+02	7.39e-02	1.45e-05	7.02e-02	6.84e-02	0.00e+00	1.60e-02	9.65e+00	2.32e+01	1.12e+01	0.00e+00
f_{15}	-4.60e-01	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	-9.99 e-01	-9.29e-01	-9.51e-01	-1

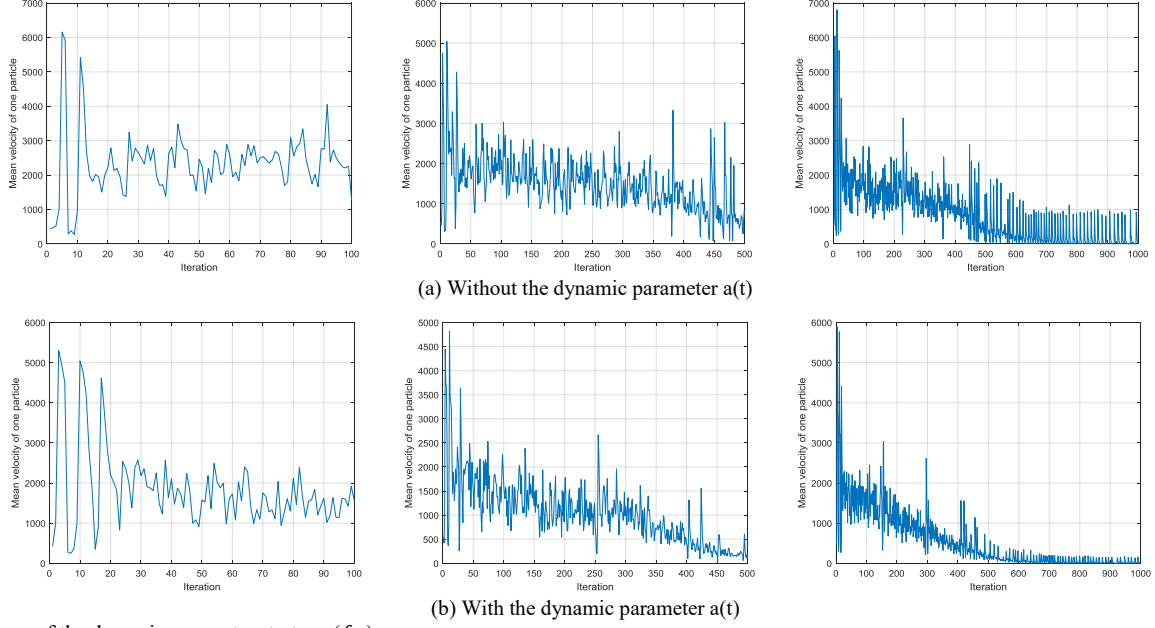


Fig. 1. Influence of the dynamic parameter strategy (f_{10})

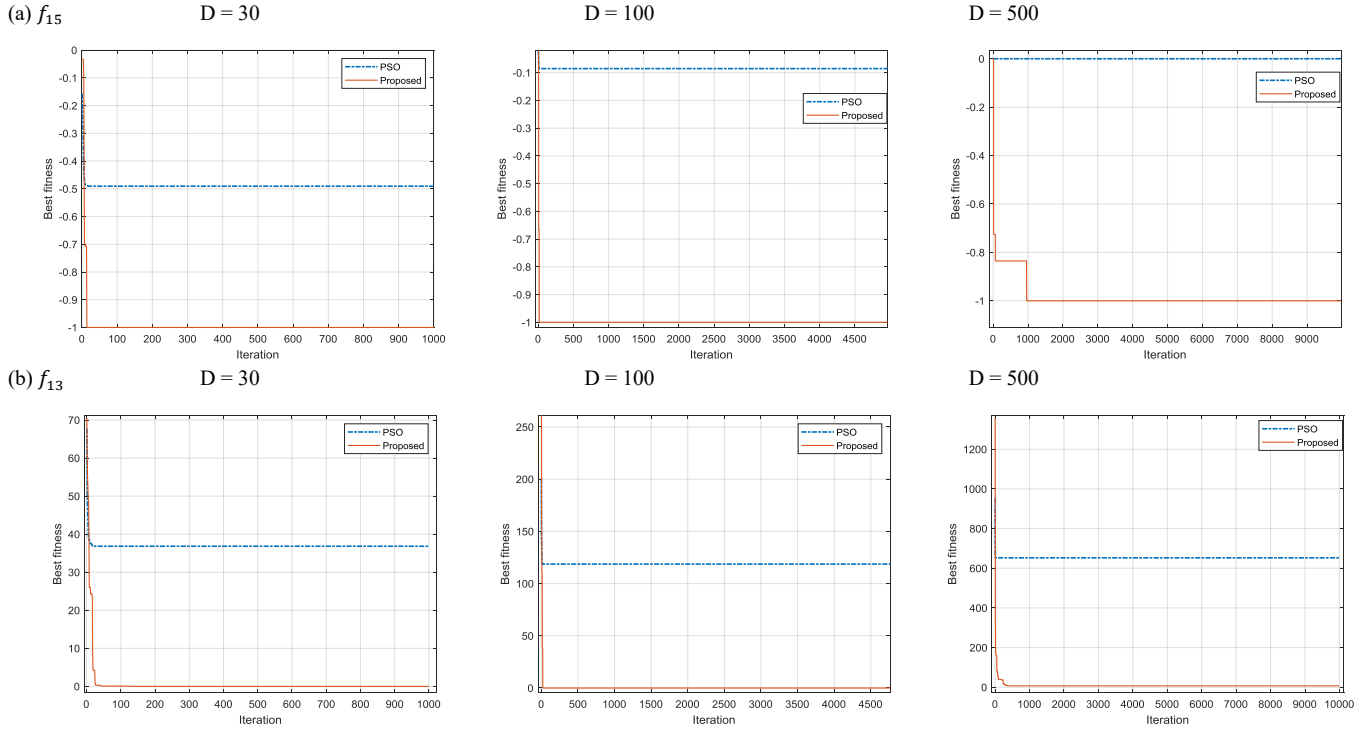


Fig. 2. Evolution of the solution with the number of iterations for different dimension sizes ($D=30, 100$ and 500)

V. CONCLUSION

Dynamic Cognitive-Social Particle swarm optimization is an effective algorithm for optimizing several benchmark functions regardless the dimension. The algorithm has shown great reliability in getting a *satisfactory* solution when it was tested on different known common functions in the literature review. The Dynamic parameters along with controlling the recursive velocity equation that was integrated in the algorithm make the convergence to the global solutions faster.

In order to better model the movement of bird or fish swarm, further investigation in finding better dynamic parameters along with clustering the set of particles will improve DCS-PSO in reaching the global solution faster.

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