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MUTRISS: A new method for material selection problems using MUltiple-TRIangles scenarios

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ABSTRACT

This paper proposes a new Multiple-criteria decision-making (MCDM) method called MUltiple-TRIangles ScenarioS (MUTRISS) with two scenarios respecting different levels of access to complete information for material selection problems. MUTRISS calculates the areas occupied by alternatives in n-dimensional space, employing analytic geometry and converting each alternative into n-edges forms. The paper applies MUTRISS to three material selection case studies, with Ti-6Al-4V, Material 4, and AISI 4140 Steel- UNS G41400 emerging as the best materials for the three examples with the highest overall scores of 0.036, 4.540 and 0.427 respectively. The results are compared with various MCDM methods through four statistical measures, including relative closeness ratio, robustness analysis, compromise ranking coefficient, and similarity degree. The measures focus on different aspects of MCDM methods in solving problems and their results. The paper concludes that MUTRISS offers a more robust and reliable approach for material selection problems compared to other MCDM methods, with the first scenario of MUTRISS being more reliable than the second scenario. The paper also emphasizes the importance of validating results in material selection problems due to the potential irreversible consequences of selecting the wrong material.

Nomenclature.

As scientific papers usually employ several symbols and notations for the vectors, matrices, and random variables. The conventions applied in this paper are displayed in (Table 1).

1. Introduction

The need to survive in the manufacturing industry has driven organizations to create high-quality, affordable products with improved performance. The development of numerous new materials in recent years has led to the replacement of the earlier available materials. Due to poor material selection, many excellent designs are never realized. For any design to be reliably manufactured, the choice of an acceptable material or material combinations is therefore very essential. Additionally, competition between industries is growing as a result of the constantly shifting customer demands, both to increase market share and to preserve the environment for coming generations. Although material selection can be made at any point during the life cycle of a product, it is often done during the initial design phase. Engineering design addresses the issue of ongoing improvement through improved

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Table 1
The list of symbols and notations.

Symbol	Description
w_j	Criteria weights of in a decision matrix
c_j	Criteria in a decision matrix
A_i	Alternatives in a decision matrix
m	Number of alternatives
n	Number of criteria
AV_i	Alternative value of i the alternative in MUTRISS scenarios
ϕ	Angle between each alternative in 360-degree angle-flat space in MUTRISS the first scenario.
θ_j	Angle between alternatives in MUTRISS the second scenario
ρ_{ki}	Spearman's rank correlation coefficient between the k th and i th MCDM method
q	Number of MCDM methods in the Spearman's rank correlation coefficient formula
RCR_κ	Relative closeness ratio
$AV_{i\kappa}$	Rank of i th alternative as the output of n th MCDM method in the relative closeness ratio formula
N_e	Number of alternatives of e th case evaluated in the computing process the relative closeness ratio
$\mathbb{N}$	Number of cases considered in the computing process the relative closeness ratio
RCR_κ^ρ	Total similarity
e_i	Entropy of the ranks of i th alternative in the (κ) MCDM methods in the RCR algorithm
RCR_κ^β	Primary results of RCR_κ
RA_k	Robustness analysis
RA_κ^β	Primary form of the RA_k in the computing process of the robustness analysis
$\mathbb{K}$	Total number of the MCDM methods analyzed in the computing process of the robustness analysis
N_e	Number of alternatives of e th case evaluated in the computing process of the robustness analysis
$\mathbb{N}$	Number of cases considered in the computing process of the robustness analysis
$\mathbb{s}_k$	Number of algorithm steps of κ the MCDM method applied on the $\mathbb{N}$ cases
ϑ_z	Ranking compromise coefficient
R_{i^*}	Distribution values of each rank for each alternative in the computing process of the ranking compromise coefficient
$V_{R_{i^*}}$	Value of each rank in the computing process of the ranking compromise coefficient
Y	Ranks' values matrix
ζ_z	Compromise degree
η_l	Similarities degree of l $^{\text{th}}$ MCDM method

design and does so using both the currently accessible materials and novel material combinations. Design engineers are faced with the dilemma of selecting the best appropriate material to fulfill the needs of a product design due to the availability of numerous materials with various qualities.

There are cases where design requirements and objectives may conflict, and it becomes necessary to prioritize certain features over others. The high accretion of several factors for product performance, including dependability, safety, strength, environmental friendliness, energy-saving qualities, and economic considerations, further increase the selection process's complexity. Design engineers constantly strive for a compromise between these factors and the demands of end customers who will pay for the product in the face of such growing complexity. In order to find the appropriate material, a systematic process must be followed; otherwise, an unsuitable design might result in a catastrophic product collapse, which can occasionally be lethal as well. In order to choose the best material for a particular product, it is necessary to compare the attributes of a limited number of materials. Nevertheless, when selecting a material for an engineering application, the designers frequently use trial-and-error techniques or rely on their expertise and experience, which may be ineffective in any given situation. The oldest and most straightforward approach for selecting an appropriate material is the conventional selection method, which is based on receiving the required information, e.g., the materials' physical properties and performance attributes, from material manufacturers, suppliers, and standards (Şensoy et al., 2019). In order to achieve consistency between design, manufacturing objectives, functions, shape, process, and materials, it is required to choose the optimal material for a given application using an organized and efficient strategy. The procedure of choosing a material for a particular component is frequently challenging and time-consuming due to the abundance of materials and the intricate interactions between many selection characteristics. According to İpek et al., (2013), selecting the appropriate material from a database with over 100,000 alternatives requires an advanced system due to the complexity of the material selection process. In general, the selection of materials for a given product embraces two main phases, including the investigation/evaluation of product requirements and the selection of the best alternative material (Mousavi-Nasab and Sotoudeh-Anvari, 2017). Due to the enormous number of available engineering materials and their diverse production technologies, the selection process can be regarded as a complex decision-making task requiring a systemic mathematical approach to simplify (Hafezalkotob and Hafezalkotob, 2015). As a result, choosing the best material for a given application typically involves choosing the best possible combination of properties rather than just one, which makes it a multi-criteria decision-making (MCDM) problem. To select an appropriate material, Findik & Turan. (2012) mentioned three main methods, including: 1. the cost per unit property method; 2. the limits on properties method, and 3. the weighted property index method, which the latter functions similar to the simple additive weighting (SAW) MCDM method. Abishini and Karthikeyan (2023) argued that MCDM methods are employed to select the optimum material from a list of candidate materials selected for the specific product requirements with conflicting multiple attributes, limitations, or preferences. MCDM methods split alternatives into several aspects, attributes, and characteristics according to the number of criteria to run the alternatives' evaluation process (Zakeri et al., 2022). Fig. 1 shows a typical material selection procedure.

Das *et al.* (2016) also described the material selection as a two-step process in which the first step embraces the alternative generation by Ashby Material Selection Chart[6], and the second step involves the application of MCDM, multi-attribute decision-making (MADM) or multi-objective decision-making (MODM) methods. Various methods for solving MCDM problems have already been developed and are employed in diverse material selection situations to provide optimal decisions. For instance, (Meng and Dong, 2022; Javaid et al., 2023; Kirişci et al., 2022; Singh et al., 2020) used outranking methods[7], while compromise ranking methods are used in (Sanghvi et al., 2021), distance-based methods[8] were employed in (Howari et al., 2023; Bhadra et al., 2022; Kamble et al., 2022; Subba & Shabbiruddin., 2022; Zakeri and Konstantas, 2022; Dhanalakshmi et al., 2020), and employed pairwise comparison methods[9] to evaluate the materials in their studies were used by (Chen et al., 2023; Rajput et al., 2022; Varghese and Karande, 2022; Mastura et al., 2022).

Despite being essentially effective at resolving material selection challenges, MCDM methods have some drawbacks. Each MCDM method may suggest a different solution for the same selection problem since they all use different mathematical procedures to choose the optimal

[6] See (Bird et al., 2018; Shah, 2014; Parate & Gupta, 2011).

[7] The Outranking methods compare all possible couples of alternatives in the decision-making matrix and determine which are preferred by systematically evaluating them regarding each criterion. The most well-known outranking method is the PROMETHEE method.

[8] Along with the pairwise comparison methods, the distance-based methods are probably the most popular MCDM methods. They derive at least one abstract alternative with the highest score, called the ideal alternative, in each criterion from the decision matrix and compare the distance of each decision's alternative with this abstract alternative. The most famous MCDM distance-based methods are TOPSIS and VIKOR.

[9] These methods use pairwise comparisons as a tool to gauge the preference of the decision alternatives with regard to the criteria of the problem. AHP is the most famous pairwise comparison method.

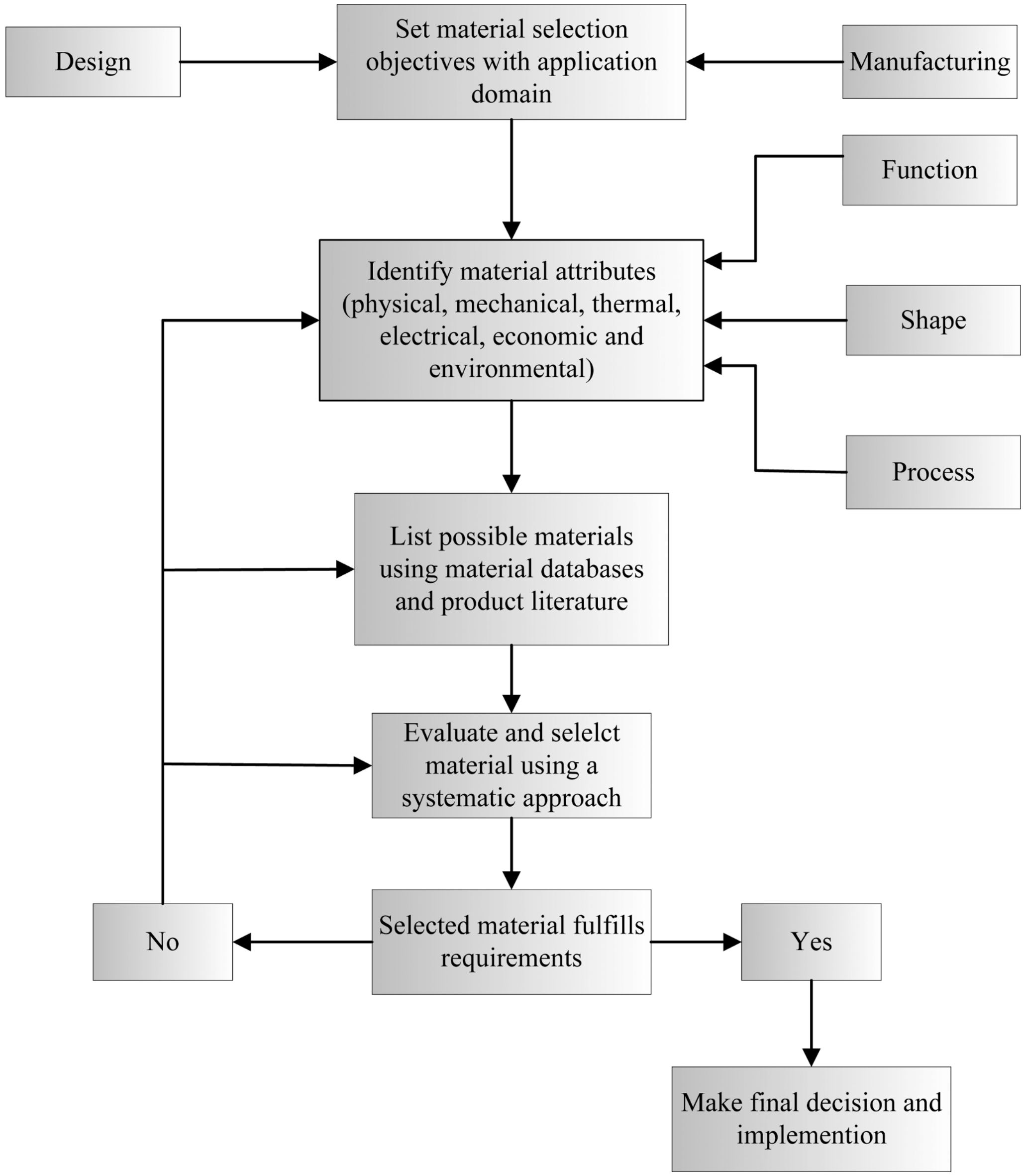

Fig. 1. A typical material selection process.

alternative (Triantaphyllou and Mann, 1989). The aforementioned variations in the outcomes produced by various MCDM methods arise as a significant issue in material selection applications where poor judgments carry high levels of risk and frequently end in unrecoverable scenarios. Differences in rankings obtained by various MCDM methods for the same material selection problem may be found in (Bhadra et al., 2022; Nguyen et al., 2022; Zhang et al., 2020a; Zhang et al., 2020b; Singh et al., 2020; Mousavi-Nasab and Sotoudeh-Anvari, 2017; Hafezalkotob and Hafezalkotob, 2015). According to Tscheikner-Gratl et al. (2017), the results of different MCDM methods might not be comparable for the same case study because of the selected score scales and consequent distributions of scores within the criteria. According to Zanakis *et al.* (1998), the four leading causes of inconsistent outcomes produced by MCDM methods are as follows: (i) different weights are used in the calculations between the methods. (ii) different algorithms take different approaches to choose the best solution; (iii) several algorithms try to scale the objectives, which has an impact on the predetermined weights; (iv) some algorithms introduce additional parameters that have an impact on the solution to be chosen. The weights of criteria and the principles that MCDM methods use for their alternative evaluation process are the most crucial components that directly influence the outcome of any MCDM method. No ranking system is aimed at explicitly taking the knowledge of the decision-makers (DMs) into account. In most MCDM methods, the primary responsibilities of DMs are limited to identifying potential alternatives with criteria values and criteria weights. Due to the DMs' ineptitude, which muddies the results, the latter has always been of severe concern. When several MCDM methods yield comparable rankings, even the numerical ratios between the alternatives do not match. When several alternatives need to be considered, and the ratio used to choose a set of alternatives is crucial, this problem becomes vital. Another issue arises when DMs need to validate their results, and the rankings produced by various MCDM methods are not comparable. In this case, statistical measurements are necessary to validate the results. It is crucial to employ a method that optimizes material selection decisions and minimizes the risk of poor selection because appropriate material selection results in improved quality and enhanced product life cycle. In contrast, inaccurate selection leads to increased design cost, lack of productivity, the poor performance of the end product, critical component damage, and, eventually, untimely product failure (Patnaik *et al.*, 2020). According to the issues mentioned above, by proposing a new MCDM method and a number of statistical measures, this paper aims to focus on the following: 1. better performance compared to other MCDM methods in ranking materials in a material selection problem, theoretically and mathematically, in the presence of decision-makers different levels of access to the complete information; 2. addressing the lack of the current MCDM algorithms' outputs validation approaches. As a novel MCDM method, MUltiple-TRIangles ScenarioS (MUTRISS) exercises analytical geometry concepts and applies the two scenarios, the proposed method seeks to offer accurate solutions that minimize the risk of poor selection in the material selection problems through robust yet simple algorithmic steps. It also offers unique processes for decision-makers with different information access levels. The benefits derived from this paper in proposing solutions for solving material selection problems address primarily the risk of wrong selection, which leads to unpredictable and irreversible consequences that are challenging to manage in most cases. The measures proposed in this paper provide assistance in validating the outputs of MCDM methods, and can be applied to the validation of results from any MCDM method. These issues are not commonly addressed in the literature, and previous studies have not provided solutions to these problems, thus making use of the contributions of this paper vital for solving material selection problems, as well as for the validation of MCDM methods in general. The remainder of this paper is organized as follows: The MUTRISS method is presented as a new MDCM method in Section 2. In Section 3, three cases of material selection problems are presented. In the fourth section, results obtained from MUTRISS scenarios and different MCDM methods are compared comprehensively. Finally, conclusions and suggestions for future research are presented in Section 5.

2. MUTRISS method

The MUTRISS method is suggested in this study to overcome the drawbacks of current MCDM methods, such as inconsistent rankings, selecting several alternatives as the best choice, and ignoring the involvement of DMs in the decision-making process. According to the level of knowledge that the DM(s) have regarding the criteria, the MUTRISS method is designed in two alternative scenarios to address material selection challenges. The descriptions below include alternatives for the materials that could be used to solve the material selection problems under consideration, and the criteria refer to the variables that DMs use to assess the potential materials.

2.1. MUTRISS the first scenario

In the first scenario, DMs lack complete information about the alternatives and criteria due to the availability of partial information about criteria weights. An MCDM method evaluates alternatives against the criteria in an n-dimensional space (where n is the number of criteria), i.e., each alternative occupies a specific space as a multidimensional shape in the n-dimensional space. This is because the Euclidean space is formed/limited by points that architect a multidimensional object. MUTRISS calculates the areas of these new shapes after converting the multidimensional shapes in the first scenario into bi-dimensional shapes surrounded by (C_j) and (C_{j+1}) dimensions where ($j + 1 \le n$), then calculates the areas of these new shapes, as shown in Fig. 2.

After the normalization process, the first scenario of MUTRISS calculates each alternative area as an n-edges shape using the concept of irregular polygons. Every mentioned irregular polygon is built on multiple triangles (Fig. 3). To calculate the area, MUTRISS estimates the area of n-triangles. The alternative with the highest corresponding value of area is selected as the best alternative, *i.e.*, the alternative which occupies the most area in the n-dimensional space is considered to be the best one.

The three-step MUTRISS algorithm is arranged according as follows:

Step 1. To keep the value of each side between 0 and 1, where

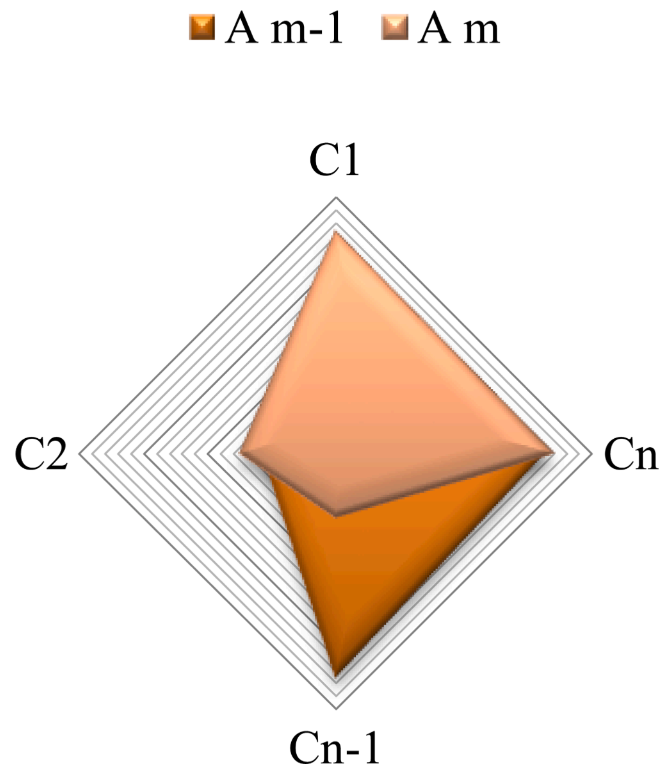

Fig. 2. Alternatives in a bi-dimensional space and as an n-edges shape.

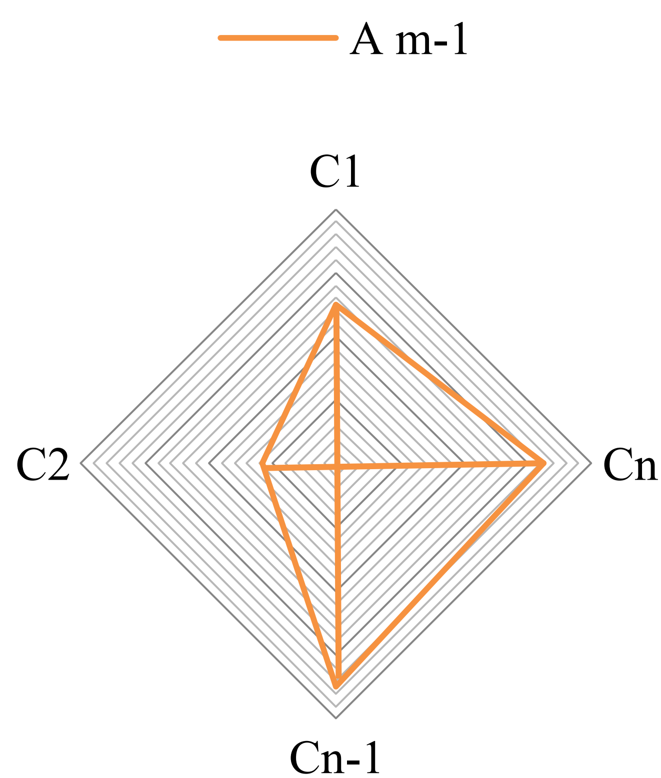

Fig. 3. Area of (A_{m-1}) alternative: each alternative includes (n) triangles.

($0 < \overline{x}_{ij} \le 1$), the first step of the proposed algorithm is the normalization of the decision matrix with respect to (Eqs. (1), 2) and converting it into a beneficial matrix, in which the highest value is favorable. In the equations, ($X = x_{ij}$) denotes the decision matrix.

For the beneficial attributes/criteria:

$$\bar{x}_{ij} = \frac{x_{ij}}{\max\limits_{1\le j\le n} x_{ij}}, i = 1, \cdots, m \tag{1}$$

For the non-beneficial (cost) attributes:

$$\bar{x}_{ij} = \frac{\min\limits_{1\le j\le n} x_{ij}}{x_{ij}}, i = 1, \cdots, m \tag{2}$$

In contrary to beneficial attributes, a lower value is favorable for non-beneficial (cost) attributes.

Step 2. Establishing weighted decision matrix using Eq. (3).

$$r_{ij} = w_j \tag{3}$$

where $w_j = \{w_1, \cdots, w_n\}$ are the weights of criteria which can be obtained from the MCDM subjective weighting methods, or can be calculated by the MCDM objective weighting methods.

Step 3. Computation of the value for the i th alternative using Eq. (4), where (ϕ) is the angle between each alternative in a 360-degree angle-flat space and ($\phi = \frac{360}{n}$).

$$AV_i = \left(\left(\sum_{j=1}^{n} r_{ij} \times r_{(i(j+1))}\right) + (r_1 \times r_n)\right) \times \sin\phi 0.5 j + 1 \le n \tag{4}$$

When (AV_i) is higher, the ranking order of the alternative is better.

2.2. MUTRISS the second scenario

When DMs have access to complete information about criteria weights, the second scenario of MUTRISS is to be adopted. Analogous to the first scenario, the second scenario is also based on calculating areas of triangles which are restricted in a numerical interval, as shown in Fig. 4.

The weight of each criterion plays the most important role in the second scenario. In a 90-degree Euclidean subspace (see Fig. 4), each weight is proportionate to the corresponding angle, where ($\sum_{j=1}^{n} w_j = 1$), ($b = x_{mn_{min}}$) and ($\sum_{j=1}^{n} \theta_j = 90^{'}, j = \{1, \cdots, n-1\}$).

Algorithm of MUTRISS the second scenario for the i th alternative is presented in the following steps:

Step 1. Normalizing the decision matrix using Eqs. (1) and (2).

Step 2. Arranging each ($\bar{x}_j$) of i th alternative in their descending order as the following equation:

$$\bar{x}_{ij} : \bar{x}_{mn_{max}} \rightarrow \bar{x}_{ij} \rightarrow \bar{x}_{mn_{min}} = \{\bar{x}_{mn_{max}}, \bar{x}_{mn_{max}-1}, \cdots, \bar{x}_{mn_{min-n}}, \bar{x}_{mn_{min}}\} \tag{5}$$

Step 3. Constructing triangles in a 90-degree angle flat space, where ($\bar{\boldsymbol{x}}_{\boldsymbol{mn_{min}}}$) is an abstract axiom that the adjacent side of a right-angled triangle is located on it, and ($\bar{\boldsymbol{x}}_{\boldsymbol{mn_{min-n}}}$) and ($\boldsymbol{b}$) are the hypotenuse and adjacent side respectively (see Fig. 4). To calculate each triangle, MUTRISS deals with trigonometric ratios.

- To calculate angle between alternatives, the second scenario uses the following equation, where the number of constructed angles is $n-1$, and which equation is developed in order to calculate each angle of the i th alternative.

$$\theta_j = w_{\bar{x}_{ij^*}} \left(w_{\bar{x}_{ij^*}-1}\right)^{-1} \left(\sum w_{\bar{x}_{ij^*}} \left(w_{\bar{x}_{ij^*}-1}\right)^{-1}\right)^{-1} \times 90, j = \{1, \cdots, n-1\} \tag{6}$$

$$\bar{x}_{ij^*} > \bar{x}_{ij^*-1}$$

For instance:

$$\theta_{n-1} = w_{\bar{x}_{mn_{min-n}}} \left(w_{\bar{x}_{mn_{min}}}\right)^{-1} \left(\sum w_{\bar{x}_{ij^*}} \left(w_{\bar{x}_{ij^*}-1}\right)^{-1}\right)^{-1} \times 90 \tag{7}$$

where $\bar{\boldsymbol{x}}_{\boldsymbol{ij^*}}$ and $\bar{\boldsymbol{x}}_{\boldsymbol{ij^*-1}}$ stands for two edges of the triangle.

With respect to ($\bar{\boldsymbol{x}}_{\boldsymbol{j}}$) and ($\boldsymbol{w_j}$), [$\theta_1, \theta_{\boldsymbol{n}-1}$] is the numerical interval of angles, where θ_1 is the angle between ($\bar{\boldsymbol{x}}_{\boldsymbol{mn_{max}}}$) and ($\bar{\boldsymbol{x}}_{\boldsymbol{mn_{max}-1}}$) while the ($\bar{\boldsymbol{x}}_{\boldsymbol{mn_{max}}}$) and ($\bar{\boldsymbol{x}}_{\boldsymbol{mn_{max}-1}}$) refer to the highest and second highest values of ($\bar{\boldsymbol{x}}_{\boldsymbol{j}}$) of the $\boldsymbol{i}$ th alternative respectively. ($\bar{\boldsymbol{x}}_{\boldsymbol{mn_{min}}}$) stands for the lowest value of ($\bar{\boldsymbol{x}}_{\boldsymbol{j}}$) of the $\boldsymbol{i}$ th alternative. In contrast to existing MCDM methods, the second scenario of MUTRISS does not analyze the weighted normalized matrix, and the weights do not make direct impact on the normalized decision matrix ($\bar{\boldsymbol{x}}_{\boldsymbol{ij}}$). However, the area of each triangle is the space where the impact of the weights becomes visible.

Step 4. The final step is calculation of overall score of the alternatives with the computation of areas alternatives occupy according to the following equation.

$$\boldsymbol{AV_i} = \sum_{j=1}^{n} \bar{\boldsymbol{x}}_{\boldsymbol{ij^*}} \bar{\boldsymbol{x}}_{\boldsymbol{ij^*-1}} \mathbf{sin}\theta_j 0.5 \tag{8}$$

In line with (AV_i), the alternatives are arranged in descending order.

The flow chart of MUTRISS and how it splits into two different scenarios are displayed in Fig. 5. The difference between the two scenarios can be found in Eqs. (8) and (4). In the first scenario, the range of angles rises until it reaches 360 degrees, when no complete information is readily available. The second case, on the other hand, results in a lesser

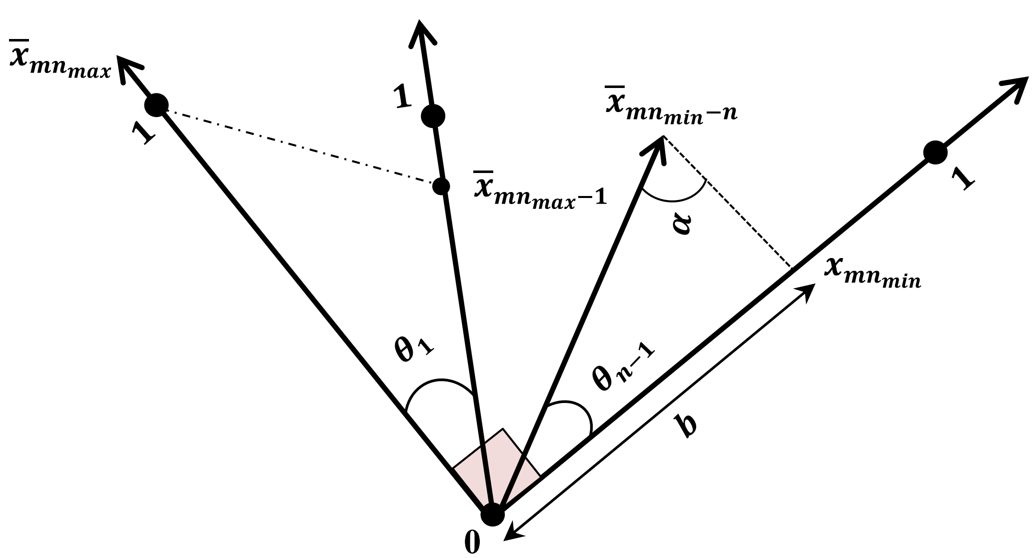

Fig. 4. Portrait of second scenario of MUTRISS.

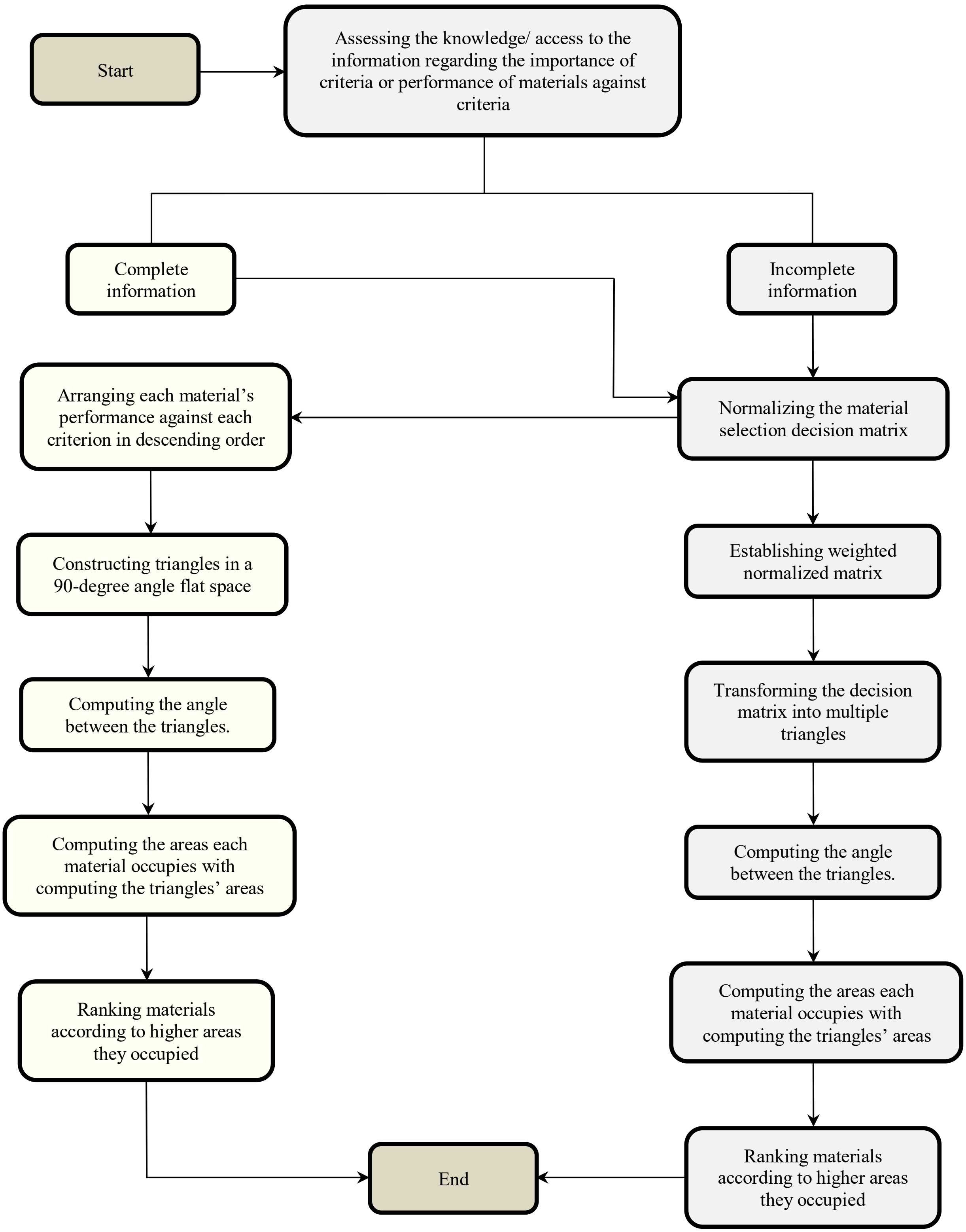

Fig. 5. Flow chart of MUTRISS process.

angle range of 90 degrees. Furthermore, due to incomplete information in the first scenario, the angles are assumed to be equal, whereas, in the second scenario, the weight determines the areas of the triangles. When DMs only have access to partial information, the first scenario can be employed; otherwise, the second scenario should be applied. As a result, DMs use the information at their disposal to calculate criteria weights more precisely than in the first scenario. The knowledge of DMs will always be ambiguous, and they frequently have imperfect knowledge, incomplete facts, or incomplete opinion. Thus, the first scenario can be effectively employed to define the problem for determining the ranking of alternatives in such circumstances, as illustrated in Fig. 6.

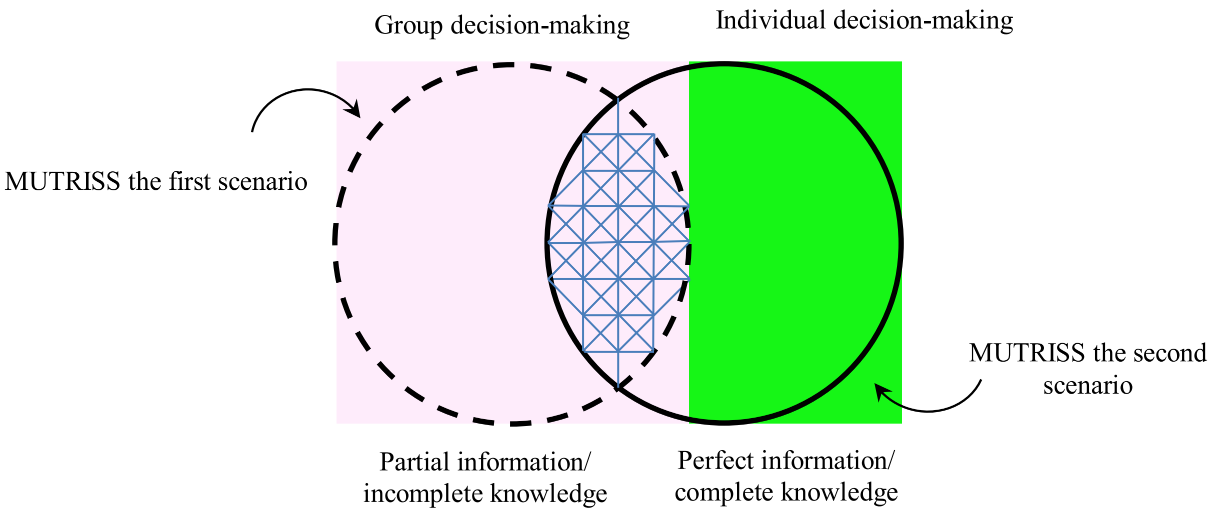

Fig. 6. Relationship between the levels of DM(s)' information/knowledge and MUTRISS scenarios.

3. Application of MUTRISS in material selection problems

In order to demonstrate the potential use of the MUTRISS method employing both scenarios, three case studies have been taken into consideration in this section. The first scenario of MUTRISS is applied to two examples, including a material selection problem for a cryogenic storage tank and a product for a high-temperature, oxygen-rich environment. The second scenario of MUTRISS is applied to select the best piston material for a given case study.

3.1. Example 1- cryogenic storage tank material selection

The first example is taken from Rao (2006) with the intention of evaluating seven alternative materials. It involves choosing the best material for a cryogenic storage tank for the transformation of nitrogen liquid while taking into account seven material selection criteria, including toughness index (TI), yield strength (YS), Young's modulus (YM), density (D), thermal expansion coefficient (TE), thermal conductivity (TC), and specific heat (SH). TI, YS, and YM are among the beneficial criteria (higher values are desirable) while the others are non-beneficial (cost) and call for lower values. In this problem, criteria weights are given as $w_{TI} = 0.28$, $w_{YS} = 0.14$, $w_{YM} = 0.05$, $w_D = 0.24$, $w_{TE} = 0.19$, $w_{TC} = 0.05$, $w_{SH} = 0.05$. The material selection decision matrix is shown in Table 2.

3.2. Example 2 – Material selection for high-temperature oxygen-rich environment

The problem of selecting the best work material for a unique product that has to be developed to operate in a high-temperature, oxygen-rich environment is the second example of material selection case study (Chatterjee et al., 2011). The problem has six alternative materials, each of which is assessed by four material selection criteria like hardness (H), machinability rating of the work material based on cutting speed (M), material cost (C), and corrosion resistance (CR). C is the only non-beneficial criterion for this problem. Criteria weights are given as $w_H = 0.2362$, $w_M = 0.2663$, $w_C = 0.3042$, $w_{CR} = 0.3042$. The material selection decision matrix is shown in Table 3.

3.3. Example 3 – piston material selection

This example focuses on a piston material selection problem. Eight criteria are considered in order to select the best material out of the following alternatives: Aluminum 2618-T61, UNS A92618, Aluminum 4032-T6, UNS A94032, Aluminum A360.0-F Die Casting Alloy, UNS A13600, Aluminum 6061-T6, UNS A96061, Gray Cast Iron, SAE G4000, UNS F10008, AISI 8660 Steel/A332, UNS G86600 and AISI 4140 Steel. The minimum value is preferred over the non-beneficial (cost) criteria of density (g/cc) and cost ($/kg). Benefit criteria that call for higher values include Knoop Hardness (HK), Yield strength (MPa), Modulus of elasticity (GPa), Specific Heat Capacity (J/g-°C), Machinability (percent), and Fatigue Strength (MPa). The decision matrix, including criteria weights and performance of alternatives against the criteria is given in Table 4. DMs have access to complete information regarding the criteria for this example. In order to determine criteria weights, logarithm methodology of additive weights (LMAW), developed by Pamučar et al. (2021), is adopted here due to its sound performance in generating stable results. Expert opinions of Table 5 are used in LMAW method for further analyses. In the process of weight determination, nine linguistic variables and their corresponding numeric values are adopted as shown in Table 6 and the corresponding final weights are given in Table 7.

4. Results and discussions

The two MUTRISS situations are now compared with other MCDM methods in a variety of ways. This section has two sub-sections since the architecture of the two scenarios differs, and they have been used with

Table 2
Decision matrix for example 1 (Rao, 2006).

Material	TI	YS	YM	D	TE	TC	SH
Al 2024-T6	75.5	420	74.2	2.8	21.4	0.37	0.16
Al 5052-O	95	91	70	2.68	22.1	0.33	0.16
SS 301-FH	770	1365	189	7.9	16.9	0.04	0.08
SS 310-3AH	187	1120	210	7.9	14.4	0.03	0.08
Ti–6Al-4V	179	875	112	4.43	9.4	0.016	0.09
Inconel 718	239	1190	217	8.51	11.5	0.3	0.07
70Cu-30Zn	273	200	112	8.53	19.9	0.29	0.06

Table 3
Data for example 2 (Chatterjee et al., 2011).

Material	H	M	C	CR
Material 1	420	25	5	0.865
Material 2	350	40	3	0.665
Material 3	390	30	3	0.745
Material 4	250	35	1	0.665
Material 5	600	30	2	0.665
Material 6	230	55	4	0.5

Table 4
Data for example 3.

Material	Density(g/cc)	Knoop Hardness (HK)	Yield strength	Modulus of elasticity	Specific Heat Capacity	Cost	Machinability	Fatigue Strength
Aluminum 2618-T61, UNS A92618	2.76	144	372	74.5	0.875	2.94	320	90
Aluminum 4032-T6, UNS A94032	2.68	150	317	78.6	0.85	2.80	70	110
Aluminum A360.0-F Die Casting Alloy, UNS A13600	2.68	97	165	71	0.963	4.11	50	150
Aluminum 6061-T6, UNS A96061	2.7	120	276	68.9	0.896	3.97	320	95
Gray Cast Iron, SAE G4000, UNS F10008	7.15	271	310	200	0.49	1.10	48	119
AISI 8660 Steel/A332, UNS G86600	7.85	220	1551	205	0.475	2	55	335
AISI 4140 Steel, UNS G41400	7.85	369	1050	205	0.561	3	65	590
Ductile Iron grade 65–45-12, UNS F33100	7.15	195	310	168	0.49	1.54	61	193

Table 5
Expert opinions for criteria weights.

Expert	Density (g/cc)	Knoop Hardness (HK)	Yield strength	Modulus of elasticity	Specific Heat Capacity	Cost	Machinability	Fatigue Strength
Expert 1	2.5	1.5	4	2	1.5	1.5	5	3.5
Expert 2	3	1.5	4	2.5	1.5	1.5	5	4.5
Expert 3	3.5	1.5	4	2.5	1	1.5	4	5
Expert 4	3	1.5	4	2	1.5	1.5	5	3

Table 6
Linguistic variables and their equal numeric values.

Linguistic variable	Absolutely low (AL)	Very low (VL)	Low (L)	Medium low (ML)	Equal (E)	Medium high (MH)	High (H)	Very high (EH)	Absolutely high (AH)
Scale value (1–5)	1	1.5	2	2.5	3	3.5	4	4.5	5

Table 7
Final criteria weights.

Criteria	Density (g/cc)	Knoop Hardness (HK)	Yield strength	Modulus of elasticity	Specific Heat Capacity	Cost	Machinability	Fatigue Strength
w_j	0.139	0.085	0.162	0.116	0.078	0.085	0.17	0.16

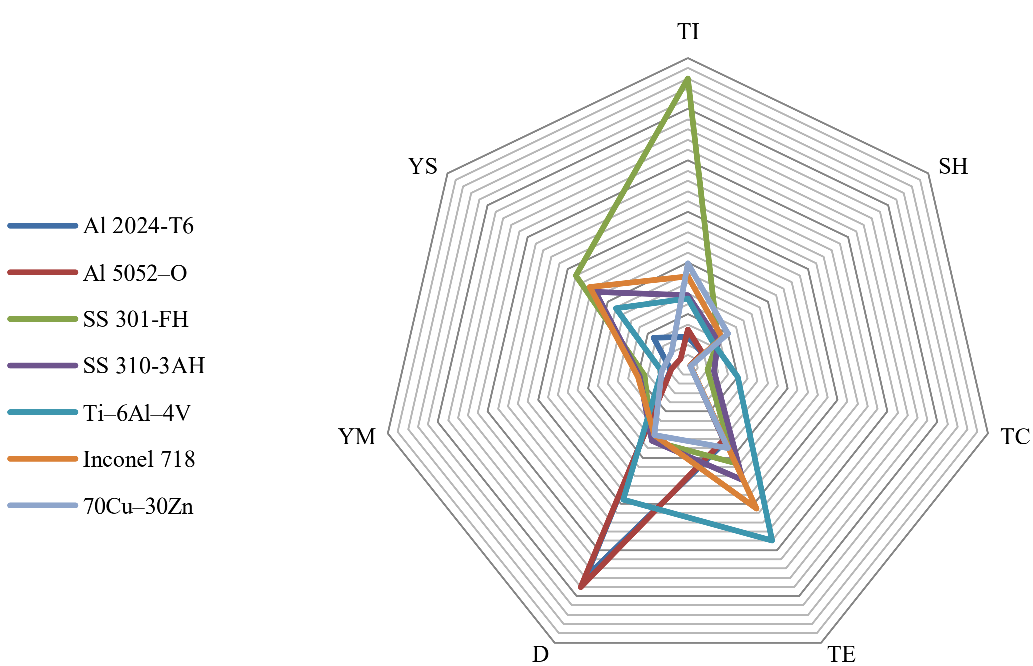

Fig. 7. Corresponding triangles for material alternatives in the first example.

various examples. Each subsection focuses specifically on one scenario and discusses each scenario's similarity, differentiation, and reliability individually.

4.1. Application of the first scenario of MUTRISS

The first scenario of MUTRISS is applied to two cases of material selection, and results are compared with COPRAS, EVAMIX, SAW, AHP, and TOPSIS methods. Figs. 7 and 8 show the intuitive structures that the MUTRISS method uses to calculate AV_i of each alternative in the aforementioned material selection problems. Several triangles that are shaped for each material are displayed in these figures. In the space of (7×7) areas, where m and n are both 7, each material has taken up an area. The larger triangle area in each figure indicates the material that should be selected as the best alternative.

MUTRISS contains data points that indicate the value of a material with regards to specific criteria used to solve material selection problems. As each data point corresponds to a different material property and criterion, the values are rescaled and adjusted to a uniform range of 0 to 1. MUTRISS method consists of a cluster of points arranged in an irregular shape which reflects the total scores of each material in the selection matrix. The shape is comprised of multiple triangles, and the size of each triangle must be computed to determine the material with the highest priority and score. A larger area in the MUTRISS denotes a material with a superior ranking. Fig. 7 displays seven points used to construct the geometries, and Ti-6Al-4V exhibits the largest area, indicating that it is the optimal material (as shown in Table 10). The shapes in Fig. 8 are constructed around four points that represent four criteria and the corresponding values of each material for those criteria. Material 4 has the largest area among the six shapes in Fig. 8, making it the most appropriate alternative for the considered problem (as given in Table 11).

According to (Eq. (4)), the following instances are provided in Table 8 and 9 to show the computation of AV_i.

Results of COPRAS, EVAMIX, SAW, AHP, TOPSIS and MUTRISS the first scenario for the considered two material selection problems are demonstrated in Tables 10 and 11.

Ranking performances of the considered MCDM methods for the two material selection case studies are inclusively shown in Figs. 9 and 10, respectively.

There is a significant variation in the overall ranks obtained by each MCDM method, as shown in Fig. 9. While SS 301-FH was chosen as the best material by COPRAS, EVAMIX, and SAW methods, it was placed in the 2nd position by the first scenario of MUTRISS and ranked 6th by TOPSIS and AHP. This indicates a considerable discrepancy in ranking order. In the second example (Table 11) and (Fig. 12), the first three ranks have been compromised. The fifth material was chosen as the best material by four MCDM methods. In contrast, the first scenario of MUTRISS and SAW showed that the fourth material (Material 4) is the best alternative and the fifth material is the second best. Less inconsistency exists between rankings in this second example than in the first. Upon closer examination of the decision matrices for the two case studies, it becomes evident that Ti-6Al-4V is unquestionably the best material for the first case study, while Material 4 is the superior choice for the second case study. This confirms the reliability and accuracy of the results generated by MUTRISS. Even though the first scenario of MUTRISS calculates the triangle-shaped areas to determine the best alternative from the decision matrix, it is a matrix-based decision model due to its first two phases. MUTRISS, a matrix-based decision-making tool, easily handles subjective and objective criteria. Both MUTRISS scenarios are more effective with physical attributes than values expressed qualitatively, just like other matrix-based MCDM methods. The first scenario in MUTRISS is built with a quick computation, and it analyses the decision matrix more straightforwardly than the second scenario. In fact, transparency does not change as the number of alternatives increases since DMs can quickly identify any errors made during the computation process. According to Chatterjee et al. (2011), it is typically recommended against using an opaque, very complicated MCDM method because any errors made during the computation process can frequently result in a high level of risk and can mislead the entire selection process.

4.2. MUTRISS the second scenario application

The results of the second scenario of MUTRISS were compared with those obtained from other MCDM methods, including the evaluation based on distance from average solution (EDAS) (Dhanalakshmi et al., 2020; Bagal et al., 2021), additive ratio assessment (ARAS) (Marichamy

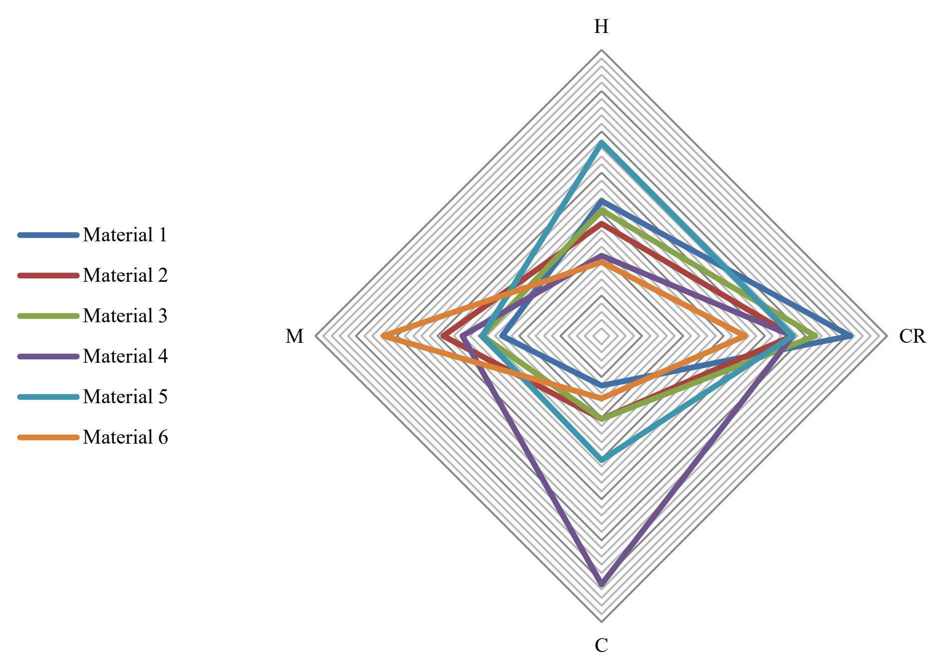

Fig. 8. Corresponding triangles for material alternatives in the second example.

Table 8
Computing AV_2 for SS 301-FH in example 1 with rank 2.

w_j	0.280	0.140	0.050	0.240	0.190	0.050	0.050		
Criteria	TI	YS	YM	D	TE	TC	SH	sum	AV_i
SS 301-FH	0.039	0.006	0.003	0.006	0.00211	0.00075	0.011	0.067	0.026

Table 9
Computing AV_4 for Material 4 in example 2 with rank 1.

w_j	0.2362	0.2663	0.3042	0.3042		
Criteria	TI	YS	YM	D	sum	AV_i
Material 4	0.017	0.037	0.067	0.030	0.151	4.540

and Babu, 2021; Goswami and Behera, 2021), MOORA, VIKOR, COPRAS, TOPSIS, and SAW, in order to cover a wider range of MCDM methods. The results of this comparison are presented in Table 12. Tables 13a and 113b.

The range of ranking distributions for each material using different MCDM methods are shown in Fig. 11.

In example 3, three methods selected the seventh alternative (AISI 4140 Steel, UNS G41400) as the best alternative. With the computation process of $(n-1)\times m$ triangles areas, MUTRISS second scenario possesses the most complex calculation amongst these MCDM methods. For instance, the following figures demonstrate the triangles that Aluminum 2618-T61, UNS A92618 has made, where $\theta_j =$ $\{14.9068, 21.7205, 11.1847, 12.1885, 8.9312, 8.7275, 12.3408\}$. These figures (Fig. 12) are extracted from T 13.

The second scenario of MUTRISS computes 56 triangular areas to determine the best alternative, whereas SAW computes the fewest computations of all the MCDM methods taken into consideration. The second scenario of MUTRISS appears as a very trustworthy method for material selection based on the performance of the investigated MCDM methods, as indicated in Table 12. In the next part, similarities between the methodologies under consideration are examined.

4.3. Performance comparison

The performance of the proposed MUTRISS method and other MCDM methods are compared in the following two sub-sections. The robustness of MCDM methods and similarity of the results are assessed using some novel statistical measures. These concepts are developed to demonstrate the benefits of MUTRISS method (for both scenarios) in order to resolve the research questions that are usually associated with MCDM methods in terms of final rankings and outputs.

Table 10
Comparative ranking for example 1.

Material	AV_i	MUTRISS the first scenario	COPRAS	EVAMIX	AHP	SAW	TOPSIS
Al 2024-T6	0.007	5	6	5	3	5	3
Al 5052–O	0.007	6	7	7	7	6	5
SS 301-FH	0.026	2	1	1	6	1	6
SS 310-3AH	0.012	4	3	4	4	4	4
Ti–6Al–4V	0.036	1	2	6	1	2	7
Inconel 718	0.012	3	4	3	5	3	1
70Cu-30Zn	0.006	7	5	2	2	7	2

Table 11
Comparative ranking for example 2.

Material	AV_i	MUTRISS the first scenario	COPRAS	EVAMIX	AHP	SAW	TOPSIS
Material 1	3.117	4	5	5	2	5	5
Material 2	2.655	5	2	2	5	3	4
Material 3	3.552	3	4	4	4	4	2
Material 4	4.540	1	3	3	3	1	3
Material 5	4.079	2	1	1	1	2	1
Material 6	2.442	6	6	6	6	6	6

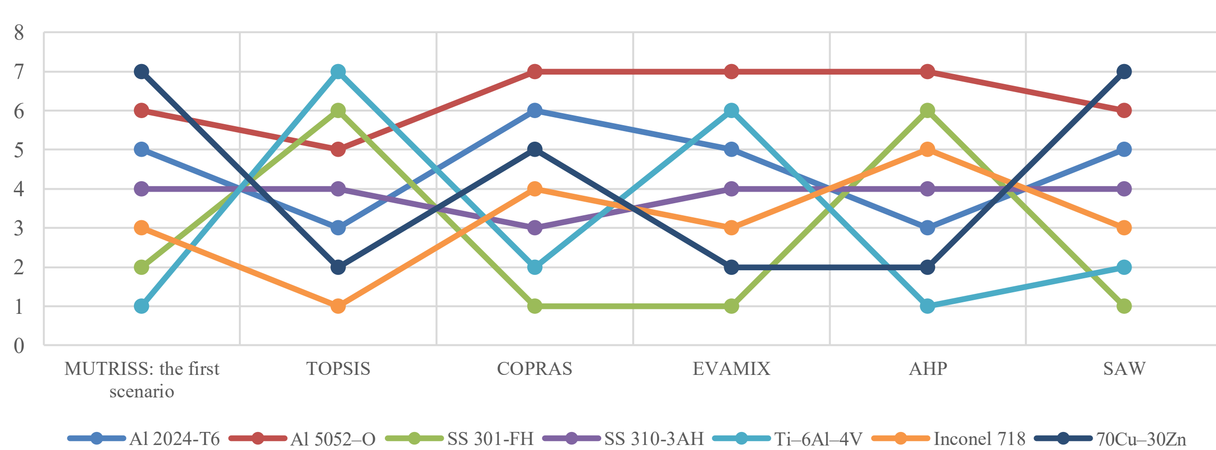

Fig. 9. Comparative ranking similarity for example 1.

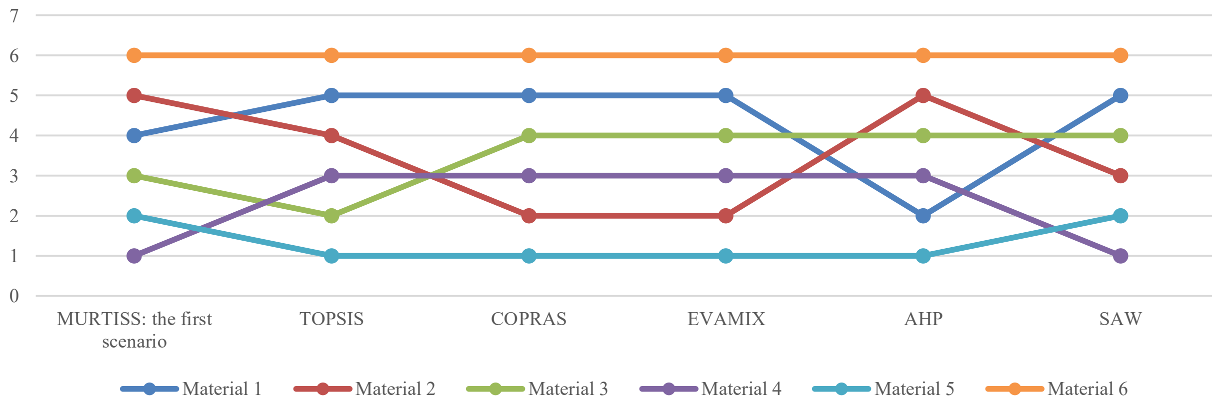

Fig. 10. Comparative ranking similarity for example 2.

Table 12
Comparative rankings for example 3.

Material	AV_i	MUTRISS' second scenario	ARAS	MOORA	EDAS	VIKOR	COPRAS	TOPSIS	SAW
Aluminum 2618-T61, UNS A92618	0.359	2	8	3	3	2	1	4	3
Aluminum 4032-T6, UNS A94032	0.248	6	5	7	7	4	5	6	7
Aluminum A360.0-F Die Casting Alloy, UNS A13600	0.241	7	7	8	8	8	8	7	8
Aluminum 6061-T6, UNS A96061	0.343	4	6	4	4	3	3	3	4
Gray Cast Iron, SAE G4000, UNS F10008	0.277	5	3	5	6	7	6	5	5
AISI 8660 Steel/A332, UNS G86600	0.346	3	4	2	2	5	4	2	2
AISI 4140 Steel, UNS G41400	0.427	1	2	1	1	1	2	1	1
Ductile Iron grade 65–45-12, UNS F33100	0.203	8	1	6	5	6	7	8	6

Table 13a
Area occupied by Alternative 1.

Criteria	Alternative 1	Weight	Angle	θ_j	Radian	Area
C_7	1	0.1700				
C_1	0.971	0.1390	1.2230	14.9068	0.2602	0.1249
C_5	0.909	0.0780	1.7821	21.7205	0.3791	0.1633
C_2	0.390	0.0850	0.9176	11.1847	0.1952	0.0344
C_6	0.374	0.0850	1.0000	12.1885	0.2127	0.0154
C_4	0.363	0.1160	0.7328	8.9312	0.1559	0.0105
C_3	0.240	0.1620	0.7160	8.7275	0.1523	0.0066
C_8	0.153	0.1600	1.0125	12.3408	0.2154	0.0039

Table 13b
Similarities of ranking between MUTRISS first scenario and COPRAS, EVAMIX, AHP, TOPSIS and SAW methods.

Example	COPRAS	EVAMIX	AHP	SAW	TOPSIS
1	0.839	0.089	−0.054	0.982	−0.696
2	0.714	0.714	0.543	1.000	0.829

4.3.1. Performance analysis of MUTRISS' first scenario

In this section, Spearman's rank correlation coefficient (Sałabun et al., 2020) has been used to assess how similar the applied MCDM methods are to the first scenario. Two novel statistical measures, called the relative closeness ratio and robustness analysis, are proposed to compare the ranks and the effectiveness of the MCDM methods.

4.3.1.1. Use of Spearman's rank correlation coefficient. To show the level of similarity between the results of the applied MCDM methods, Spearman's rank correlation coefficient is utilized here. Spearman's rank correlation coefficient is a measurement tool for computing similarity between two sets of rankings. Average similarities between the k th MCDM method and other MCDM methods is calculated using Eqn. (9), where (q) is the number of MCDM methods, and the Spearman's rank correlation coefficient (ρ_{ki}) between k th and i th MCDM methods can be obtained using Eqn. (10), where (m) stands for number of alternatives and (d_i) is the difference between two MCDM methods (Kou et al., 2012).

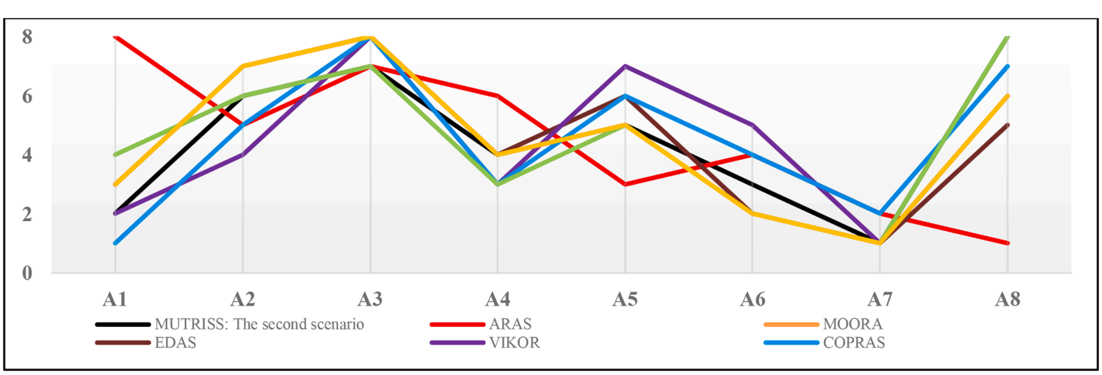

Fig. 11. Comparative ranking similarity for example 3.

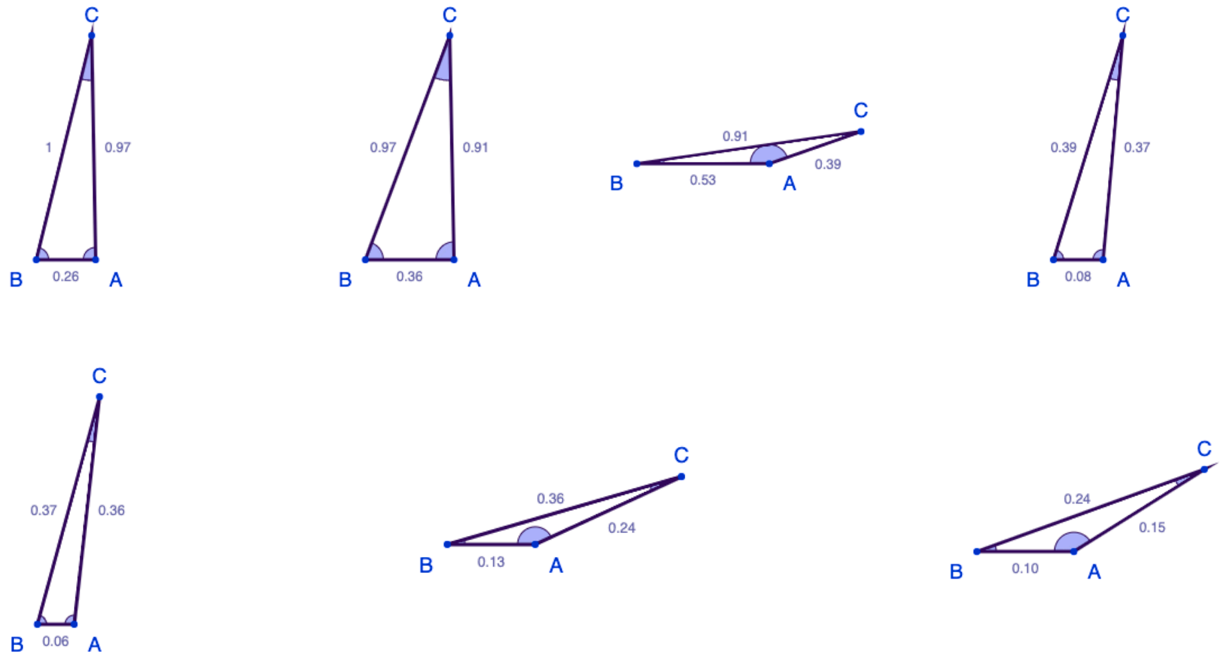

Fig. 12. Different triangles occupied by Aluminum 2618-T61, UNS A92618.

Table 14
Spearman's rank correlation coefficient between different MCDM methods.

Example	MUTRISS first scenario	COPRAS	EVAMIX	AHP	SAW	TOPSIS
1	0.2321	0.3821	0.2250	0.0036	0.3036	−0.2679
2	0.5600	0.5943	0.5714	0.6171	0.5714	0.6286

$$\rho_k = \frac{1}{q-1}\sum_{i=1, i\neq k}^{q} \rho_{ki}, k = 1, 2, \cdots, q \tag{11}$$

Where

$$\rho_{ki} = 1 - \frac{\sum (d_i)^2}{m(m^2-1)}, -1 \leq \rho_{ki} \leq 1 \tag{12}$$

Higher agreement amongst the MCDM methods is indicated by a larger value of (ρ_{ki}). In the two examples offered, similarities between the MUTRISS first scenario and other MCDM methods are shown in T 13. According to this table, results of the first scenario are the most comparable to those of the SAW method than other MCDM methods. Table 14 shows the Spearman's rank correlation coefficient for each MCDM method with the first scenario of MUTRISS.

$$RCR_\kappa = 1 - \sum_{i=1}^{m} RCR_\kappa^\beta \left(\sum_{1}^{\kappa} \frac{AV_{i\kappa}}{AV_{i1}} + \sum_{1}^{\kappa} \frac{AV_{i\kappa}}{AV_{i1}} \right) \left(\sum_{i=1}^{m} \sum_{i=1}^{m} RCR_\kappa^\beta \left(\sum_{1}^{\kappa} \frac{AV_{i\kappa}}{AV_{i1}} + \sum_{1}^{\kappa} \frac{AV_{i\kappa}}{AV_{i1}} \right) \right)^{-1} \tag{13}$$

4.3.1.2. Relative closeness ratio. The relative closeness ratio (*RCR*), represented as (RCR_κ), has been defined to analyze the similarity between each MCDM method in more than one example. *RCR* adheres pairwise comparison logic (Table 15) and adopts Shannon entropy to compute irregularities in the ranking process for each method additionally, where $AV_{i\kappa}$ is the rank of the i th alternative as the output of the n th MCDM method, (k) represents the number of MCDM methods

Table 15
Pairwise comparison of the output of each method for i th alternative.

A_i	MCDM method$_1$	⋯	MCDM Mthod$_\kappa$
MCDM method$_1$	1	⋯	$\frac{AV_{i1}}{AV_{i\kappa}}$
⋮	⋮	1	⋮
MCDM method$_\kappa$	$\frac{AV_{i\kappa}}{AV_{i1}}$	⋯	1

Table 16
Comparison between outputs of each MCDM method to computeRCP_κ

MCDM method	A_1	⋯	A_m
MCDM method$_1$			
⋮			
MCDM method$_\kappa$			

analyzed in the process, where $i = 1, 2, \cdots, m$, and $\kappa \in \{1, 2, \cdots\}$. The following equations illustrate (RCR_κ) process, where ($0 \leq RCR_\kappa \leq 1$), N_e stands for the number of alternatives of e th case evaluated in the process, C_e signifies the number of alternatives of e th case evaluated in the process, $\mathbb{N}$ states the number of cases considered in the process, and RCR_κ^ρ symbolizes total similarity. Tables 16-18.

where

$$e_i = -\frac{1}{\log\kappa} \sum_{i=1}^{m} \left(\sum_{1}^{\kappa} \frac{AV_{i\kappa}}{AV_{i1}} + \sum_{1}^{\kappa} \frac{AV_{i\kappa}}{AV_{i1}} \right) \ln \left(\sum_{1}^{\kappa} \frac{AV_{i\kappa}}{AV_{i1}} + \sum_{1}^{\kappa} \frac{AV_{i\kappa}}{AV_{i1}} \right) \tag{14}$$

Table 17
Example 1: Comparison between outputs of each MCDM method in each alternative, and RCP_κ

MCDM method	Al 2024-T6	Al 5052-O	SS 301-FH	SS 310-3AH	Ti-6Al-4V	Inconel 718	70Cu-30Zn	RCP_κ
MUTRISS	12.57	12.1	20.33	12.08	22.31	13.68	17.47	0.9923
TOPSIS	13.3	12.41	28.67	12.08	25.88	21.45	16.47	0.9990
COPRAS	13.1	12.16	20.33	12.42	16.12	14.55	14.93	0.7056
EVAMIX	12.57	12.16	20.33	12.08	23.02	13.68	16.47	0.9988
AHP	13.3	12.16	28.67	12.08	22.31	16.05	16.47	0.6215
SAW	12.57	12.1	20.33	12.08	16.12	13.68	17.47	0.6829

Table 18
Example 2: Comparison between outputs of each MCDM method in each alternative, and RCP_κ

MCDM method	Material 1	Material 2	Material 3	Material 4	Material 5	Material 6	RCP_κ
MUTRISS	12.7	14.12	12.5	17.33	14	12	0.5987
TOPSIS	12.95	13.18	14.17	14.67	13	12	0.9029
COPRAS	12.95	14.47	12.58	14.67	13	12	0.8912
EVAMIX	12.95	14.47	12.58	14.67	13	12	0.6730
AHP	16.1	14.12	12.58	14.67	13	12	0.9342
SAW	12.95	12.95	12.58	17.33	14	12	1.0000

$$RCR_\kappa^\beta = 1 - e_i\left(\sum_{i=1}^{m}(1-e_i)\right)^{-1} \tag{15}$$

And the total similarity is computed as follows (Eq. (14)).

$$RCR_\kappa^\rho = \left(1 - \left(\mathbb{N}^{-1}\sum_{1}^{\mathbb{N}} RCR_\kappa \frac{N_e}{C_e}\right)\left(\sum_{1}^{\mathbb{N}} \mathbb{N}^{-1}\sum_{1}^{\mathbb{N}} RCR_\kappa \frac{N_e}{C_e}\right)^{-1}\right) \times 100 \tag{16}$$

In the *RCR* algorithm, (e_i) stands for the entropy of the ranks of the *i* th alternative in the (κ) MCDM methods, and (RCR_κ^β) indicates the primary results of (RCR_κ). RCR_κ value of each method for each example is shown in Tables 19 and 21 respectively. These tables signify that MUTRISS shows more resemblance to SAW and EVAMIX methods than other MCDM methods employed in this paper. Total similarity of each MCDM method for the adopted two examples is given in Table 19. Figs. 13 and 13 compare the total difference and the degree to which the results of each MCDM method are similar to those of other MCDM methods. TOPSIS and AHP have the biggest difference, making their outputs the least reliable among the considered MCDM methods while MUTRISS has the least difference, as shown in Fig. 14.

4.3.1.3. Robustness analysis. To analyze robustness of the results of a specific MCDM method on more than one case in comparison with other MCDM methods, an analytical technique called *robustness analysis* (RA_k) is proposed in this paper. Larger value of RA_k indicates more robustness of an MCDM method in its algorithm and process. Equations of RA_k are as follows:

$$RA_\kappa^\beta = \left(\sum_{1}^{k} RCR_\kappa\left(\mathbb{N}\sum N_e\right)^{-1}\right)\left(\sum_{1}^{k}\left(\sum_{1}^{k} RCR_\kappa\left(\mathbb{N}\sum N_e\right)^{-1}\right)\right)^{-1} \tag{17}$$

$$RA_k = \left(RA_\kappa^\beta - \left(\sum_{1}^{k} RCR_\kappa \mathbb{N}(\mathbb{K})^{-1}\right)\right)(\mathbb{s}_k)^{-1} \tag{18}$$

(RA_κ^β) expresses the primary form of RA_k.

($\mathbb{K}$) stands for total number of MCDM methods under consideration.

(N_e) stands for the number of alternatives of e th case evaluated in the process.

($\mathbb{N}$) states number of cases considered in the process.

($\mathbb{s}_k$) is the number of algorithm steps of κ th MCDM method applied for $\mathbb{N}$ cases.

Results of the robustness analysis for each MCDM method is exhibited in Table 20.

When compared to the other MCDM methods, it is perceived that simpler algorithms of the MUTRISS and SAW methods demonstrated greater robustness in the decision-making processes, as shown in Table 20.

Table 19
Total similarity between considered MCDM methods.

	MUTRISS	COPRAS	EVAMIX	AHP	SAW	TOPSIS
RCP_κ^ρ	16.472	15.703	17.183	15.110	16.376	19.156

4.3.2. Performance analysis of MUTRISS second scenario

A piston material selection problem has been addressed using the second scenario of MUTRISS method. Compromise ranking coefficient, compromise degree, and similarities degree have been introduced in order to evaluate the ranking performance of the considered MCDM methods and also to measure similarities between them. In order to fully understand the distinct conditions in terms of access to information, the first scenario has purposefully not been adopted to this case study.

4.3.2.1. Compromise ranking coefficient. In order to compare the performances of different MCDM methods when they are applied to the same case study, a new performance indicator called the compromise ranking coefficient is introduced here. This approach works by analyzing the correlation between the derived ranks from various MCDM methods.

The following steps demonstrate the calculation process of this coefficient. Higher value of compromise ranking coefficient shows better performance of an MCDM method.

Step 1. Establishing the comparative ranking matrix.

Step 2. Compute distribution of ranks for each alternative generated by the MCDM methods (as shown in Table 21).

Step 3. Compute the value of each rank for each alternative using Eqn. (17), where R_{i^*}, $V_{R_{i^*}}$, and m stand for the distribution values of each rank for each alternative, the value of each rank, and number of the problem alternatives, respectively.

$$V_{R_{i^*}} = \frac{R_{i^*}}{m} \times 100, \quad i^* = \{1, 2, \ldots, m\} \tag{19}$$

Step 4. Convert the comparative ranking matrix into the rank value matrix (as shown in Table 22) which is constructed based on m alternatives and l MCDM methods, where $l \in z$ and $z = \{1, 2, \cdots, l\}$, where Y stand for the ranks' values matrix.

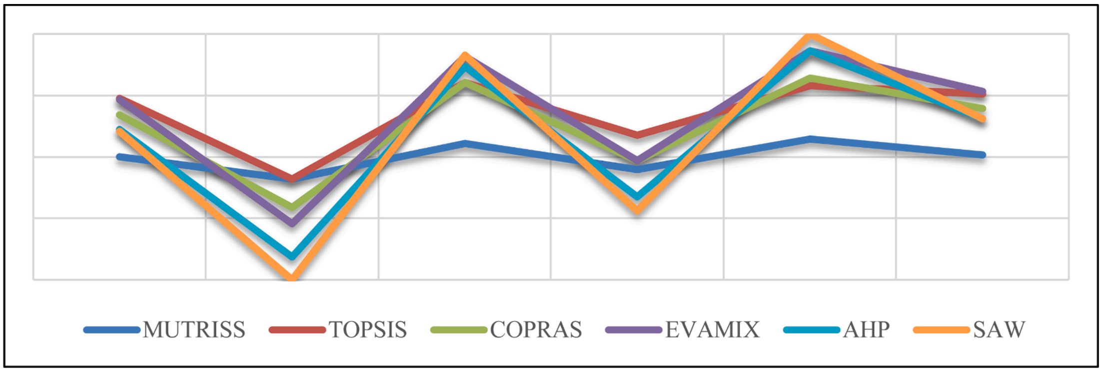

Fig. 13. Similarity comparison of each MCDM method against other MCDM methods.

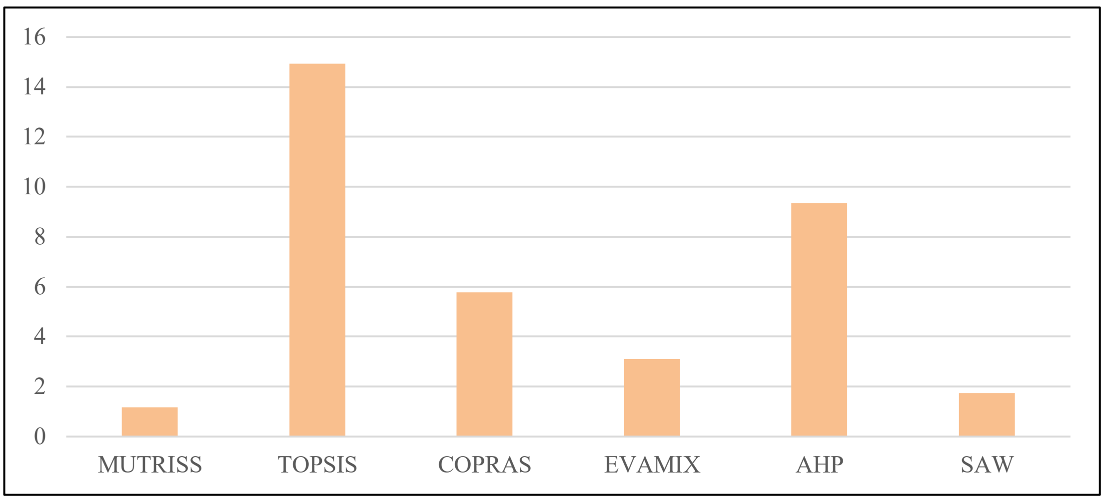

Fig. 14. Total difference between results of each MCDM method.

Table 20
Robustness analysis of MCDM methods applied to the material selection problems.

	MUTRISS	COPRAS	EVAMIX	AHP	SAW	TOPSIS
RA_k^{β}	0.1590	0.1596	0.1671	0.1555	0.1682	0.1901
RA_k	0.1237	0.0414	0.0355	0.0130	0.1309	0.0634

$$Y = V_{R_i^* iz}; i = \{1, 2, \cdots, m\}, z = \{1, 2, \cdots, l\} \tag{20}$$

Step 5. Compute the compromise ranking coefficients of Table 23 using Eqn. (19). Higher value of ϑ_z indicates better performance.

$$\vartheta_z = \sum_{i=1}^{m} V_{R_i^* iz} \left(\sum_{i=1}^{m} \max_{1\le z\le l} V_{R_i^* iz} \right)^{-1} \times 100 \tag{21}$$

4.3.2.2. The similarity degree. One of the by-products of compromise ranking coefficient is the compromise degree which indicates similarities between MCDM methods (see Table 24). The compromise degree is calculated using the following equation, where ζ_z indicates compromise degree.

$$\zeta_z = \sum_{i=1}^{m} V_{R_i^* iz} \left(\max_{1\le z\le l} V_{R_i^* iz} \right)^{-1} \tag{22}$$

The following equation is developed to calculate the similarity between k th and l th MCDM methods, where η_l is the similarity degree of l th method.

$$\eta_l = \sum_{z=1}^{l} \sqrt{(\zeta_k - \zeta_z)^2}, \quad k, l \in z \tag{23}$$

$$\eta_{l,k} = \frac{\eta_l}{\eta_k}$$

Table 21
Rank distribution for alternative materials for Example 3.

Material	First rank	Second Rank	Third Rank	Fourth Rank	Fifth Rank	Sixth Rank	Seventh Rank	Eighth Rank
Aluminum 2618-T61, UNS A92618	1	2	3	1	0	0	0	1
Aluminum 4032-T6, UNS A94032	0	0	0	1	2	2	3	0
Aluminum A360.0-F Die Casting Alloy, UNS A13600	0	0	0	0	0	0	3	5
Aluminum 6061-T6, UNS A96061	0	0	3	4	0	1	0	0
Gray Cast Iron, SAE G4000,UNS F10008	0	0	1	0	4	2	1	0
AISI 8660 Steel/A332, UNS G86600	0	4	1	2	1	0	0	0
AISI 4140 Steel, UNS G41400	6	2	0	0	0	0	0	0
Ductile Iron grade 65–45-12, UNS F33100	1	0	0	0	1	3	1	2

Table 22
Rank values for alternative materials for Example 3.

Material	The First rank	The Second Rank	The Third Rank	The Fourth Rank	The Fifth Rank	The Sixth Rank	The Seventh Rank	The Eighth Rank
Aluminum 2618-T61, UNS A92618	12.5	25	37.5	12.5	0	0	0	12.5
Aluminum 4032-T6, UNS A94032	0	0	0	12.5	25	25	37.5	0
Aluminum A360.0-F Die Casting Alloy, UNS A13600	0	0	0	0	0	0	37.5	62.5
Aluminum 6061-T6, UNS A96061	0	0	37.5	50	0	12.5	0	0
Gray Cast Iron, SAE G4000, UNS F10008	0	0	12.5	0	50	25	12.5	0
AISI 8660 Steel/A332, UNS G86600	0	50	12.5	25	12.5	0	0	0
AISI 4140 Steel, UNS G41400	75	25	0	0	0	0	0	0
Ductile Iron grade 65–45-12, UNS F33100	12.5	0	0	0	12.5	37.5	12.5	25

Table 23
Compromise ranking coefficients for Example 3.

Compromise ranking coefficient value	MUTRISS' second scenario	ARAS	MOORA	EDAS	VIKOR	COPRAS	TOPSIS	SAW
ϑ_z	75	40.625	90.625	87.5	68.75	56.25	78.125	100

Table 24
Compromise degree of each MCDM method in Example 3.

Material	MUTRISS second scenario	ARAS	MOORA	EDAS	VIKOR	COPRAS	TOPSIS	SAW
Aluminum 2618-T61, UNS A92618	0.67	0. 33	1.00	1.00	0.67	0.33	0.33	1.00
Aluminum 4032-T6, UNS A94032	0.67	0.67	1.00	1.00	0.33	0.67	0.67	1.00
Aluminum A360.0-F Die Casting Alloy, UNS A13600	0.60	0.60	1.00	1.00	1.00	1.00	0.60	1.00
Aluminum 6061-T6, UNS A96061	1.00	0.25	1.00	1.00	0.75	0.75	0.75	1.00
Gray Cast Iron, SAE G4000, UNS F10008	1.00	0.25	0.25	0.50	0.25	0.50	1.00	1.00
AISI 8660 Steel/A332, UNS G86600	0.25	0.50	1.00	1.00	0.25	0.50	1.00	1.00
AISI 4140 Steel, UNS G41400	1.00	0.33	1.00	1.00	1.00	0.33	1.00	1.00
Ductile Iron grade 65–45-12, UNS F33100	0.67	0.33	1.00	0.33	1.00	0.33	0.67	1.00
ζ_z	5.85	3.27	7.25	6.83	5.25	4.42	6.02	8.00

When $\eta_{l,k}$ value is closer to 1, MCDM methods are more similar; if $\eta_{l,k} = 1$, it indicates exactly the same results by the considered MCDM methods.

Table 24 reveals that the MUTRISS second scenario, MOORA, EDAS, VIKOR, TOPSIS, and SAW methods have a compromise ranking coefficient of 1, signifying that they have designated AISI 4140 Steel, UNS G41400 as the best material in case study 3. On the other hand, the ARAS and COPRAS methods have much lower values (0.33) for the same, resulting in smaller values of ζ_z, i.e., 3.27 and 4.42, respectively. Notably, MUTRISS second scenario, along with the other methods, demonstrates significantly greater values of ζ_z. While compromise degree is intended to estimate similarities between MCDM methods for one decision-making problem, relative closeness ratio requires more than two cases to yield the similarities. The highest value of compromise ranking coefficient is observed for SAW method, which reveals that SAW method performs better when selecting the best material for Example 3, as shown in Table 24. When it came to solving the piston material selection problem, MOORA and EDAS methods performed well. When solving Example 3, the second scenario of MUTRISS and TOPSIS produced comparatively similar results. The compromise ranking coefficient indicates how well the MCDM methods perform when ranking alternatives only.

Tables 25 and 26 respectively show the similarity between outputs of the MCDM methods. Table 26 indicates that the second scenario of MUTRISS and TOPSIS are quite similar having $\eta_{l,k}$ value of 1, while it is less similar to ARAS and SAW methods. Utilizing the similarity degree, the mentioned similarity could be also observed in (Table 25) in which $\eta_{MUTRISS' \text{ second scenario}}, \eta_{TOPSIS} = 9.32$ where ARAS and SAW are illustrating the most dissimilar results. In addition to TOPSIS, the output of MUTRISS' second scenario is comparable to VIKOR and EDAS methods. Similar to TOPSIS, VIKOR and EDAS are the distance-based MCDM method. Although MUTRISS' second scenario follows different philosophy compared to the distance-based MCDM methods, the comparisons, as provided in Tables 25 and 26 indicate the similar behavior in generating rankings.

4.4. Discussions

The success of any engineering or manufacturing project heavily relies on proper material selection, and the chances of selecting a suitable material for the application can be enhanced by adopting a systematic approach, resulting in improved product performance,

Table 25
Similarity degree of MCDM methods for Example 3.

MCDM method	MUTRISS second scenario	ARAS	MOORA	EDAS	VIKOR	COPRAS	TOPSIS	SAW	η_l
MUTRISS' second scenario	0	2.58	1.40	0.98	0.60	1.43	0.17	2.15	9.32
ARAS	2.58	0	3.98	3.57	1.98	1.15	2.75	4.73	20.75
MOORA	1.40	3.98	0	0.42	2.00	2.83	1.23	0.75	12.62
EDAS	0.98	3.57	0.42	0	1.58	2.42	0.82	1.17	10.95
VIKOR	0.60	1.98	2.00	1.58	0	0.83	0.77	2.75	10.52
COPRAS	1.43	1.15	2.83	2.42	0.83	0	1.60	3.58	13.85
TOPSIS	0.17	2.75	1.23	0.82	0.77	1.60	0	1.98	9.32
SAW	2.15	4.73	0.75	1.17	2.75	3.58	1.98	0	17.12

Table 26
Similarities between MCDM methods for Example 3.

MCDM method	ARAS	MOORA	EDAS	VIKOR	COPRAS	TOPSIS	SAW
MUTRISS' second scenario	0.45	0.74	0.85	0.89	0.67	1.00	0.54

reliability, and longevity. In this article, MUTRISS is proposed as a new MCDM method to address two primary concerns in material selection problems. Firstly, differentiation between the results obtained using various MCDM methods is aimed for. Secondly, existing gaps in the validation of MCDM results, which are often assessed using conventional approaches like sensitivity analysis, Spearman correlation, or comparing results with those of other MCDM methods to achieve a global consensus on rankings, are addressed.

In most cases, the consequences of poor material selection are complex and challenging to manage. Suboptimal performance and product failure can result from incorrect material selection, which can increase production, transportation, and maintenance costs. Safety is also a critical consideration, with materials having different properties like flammability and toxicity, making material selection crucial to ensure both product and user safety. Selecting the right material can increase the lifespan of any product, while a wrong selection can lead to premature product failure. The examples above prove the importance of using a proper process and tool for evaluating the material and validating the results.

Validating the results of MCDM methods in solving decision-making problems requires real-world experiments. However, mathematical tools are commonly used to verify the applicability of an MCDM method and confirm its results. Existing validation approaches have fundamental problems that need to be addressed. Comparing the results of an MCDM method with others is a fundamental step in validating its output. Sensitivity analysis can also confirm the final results of MCDM methods by demonstrating their robustness (Mukhametzyanov and Pamucar, 2018). However, a significant issue with sensitivity analysis is adjusting the weights based on subjective judgments. Spearman's rank correlation coefficient is another tool that can be used to verify the final findings, but it ignores the nature and structure of cases. To address these issues, this research introduced Robustness analysis and Relative closeness ratio techniques to analyze the performance of MCDM methods using several case studies. A coefficient known as the compromise ranking coefficient is also devised to quantify the relative performances of MCDM methods used to solve the same problem. It evaluates the correlation between the various rankings produced by other MCDM methods to determine their relative performances.

The paper addresses the decision-making paradox by focusing on various aspects of an efficient MCDM method for solving material selection problems, such as reliability, robustness, and transparency. The assumption was that a method possessing these three attributes could be considered a proper tool for solving material selection problems. A fourth aspect were added to the equation to complete the measurement for the best tool: the ability to coverage of DMs' different levels of access to perfect information. By running the comparison through different statistical tools on three different material selection problems, MUTRISS showed superiority over other MCDM methods in possessing the mentioned properties, which makes it a proper tool to use for solving material selection problems.

5. Conclusions and future research propositions

The MUTRISS method is proposed as a new MCDM approach for material selection problems. It involves two scenarios based on geometrical logic in a multi-dimensional space to provide accurate rankings. The first scenario applies when incomplete information is available, while the second requires complete information. MUTRISS employs triangles to represent material values based on criteria and uses the area of the shapes formed by these triangles to prioritize materials. The angles between the triangles are computed dynamically in the second scenario, while they are assumed to be equal in the first scenario. The MUTRISS method is compared to eight MCDM methods and found to be more transparent and less complex. Three examples validate the two scenarios and their outcomes are compared to other MCDM methods using various metrics including the proposed relative closeness ratio, robustness analysis, ranking compromise coefficient, similarity degree, and Spearman's rank correlation coefficients. The results show that MUTRISS is similar to SAW, EVAMIX, TOPSIS, and EVAMIX in different scenarios. The robustness analysis shows that SAW and MUTRISS offer more reliable results. However, a limitation of MUTRISS is its inability to handle uncertain variables, which could be addressed with the use of fuzzy geometry. Future research should examine the applicability of MUTRISS to other decision-making problems and explore the integration of environmental and social criteria. It is also recommended to expand MUTRISS to evaluate alternatives in multiple dimensions and integrate it with group decision-making. The accuracy of MUTRISS relies on accurate weight inputs and further feedback on its functioning is suggested.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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